Scattering Amplitudes in Theories of Compactified Gravity

Dennis Foren (he/they)

July 24th, 2020

Publications & Dissertation

- NOT Relevant -

"Direct Search Implications for a Custodially-Embedded Composite Top"
[Phys. Rev. D 94, 014002] [arXiv:1605.06088]

"Colorphilic Spin-2 Resonances in the LHC Dijet Channel"
[Physica Scripta 93 11 115301, 2018]
[arXiv:1706.02502]

- Relevant -

"Scattering Amplitudes of Massive Spin-2 Kaluza-Klein States Grow Only as $\mathcal{O}(s)$"
[Phys. Rev. D 101, 055013] [arXiv:1906.11098]

"Sum Rules for Massive Spin-2 Kaluza-Klein Elastic Scattering Amplitudes"
[Phys. Rev. D 100, 115033] [arXiv:1910.06159]

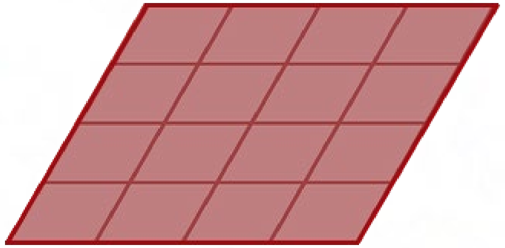
"Massive Spin-2 Scattering Amplitudes in Extra-Dimensional Theories"
[Phys. Rev. D 101, 075013] [arXiv: 2002.12458]

"Scattering Amplitudes in Theories of Compactified Gravity"
[Dissertation]

Motivating the Randall-Sundrum 1 (RS1) Model

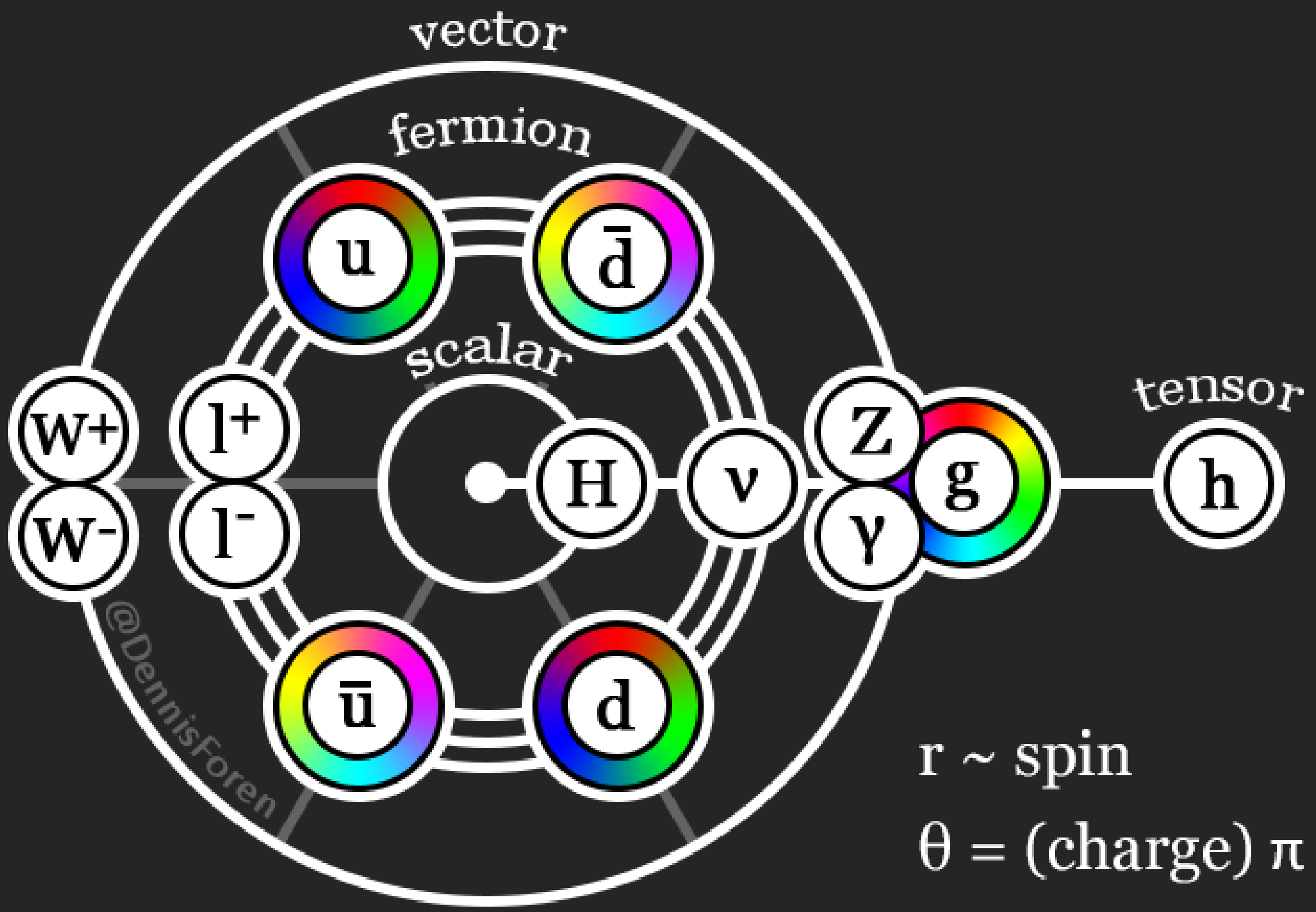
The Standard Model + Gravity

- Usual 4D Spacetime -



coordinates: $x^\mu = (x^0 , x^1 , x^2 , x^3)$

- Standard Model + Gravity -



The Standard Model + Gravity

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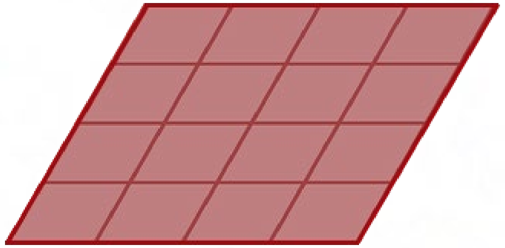

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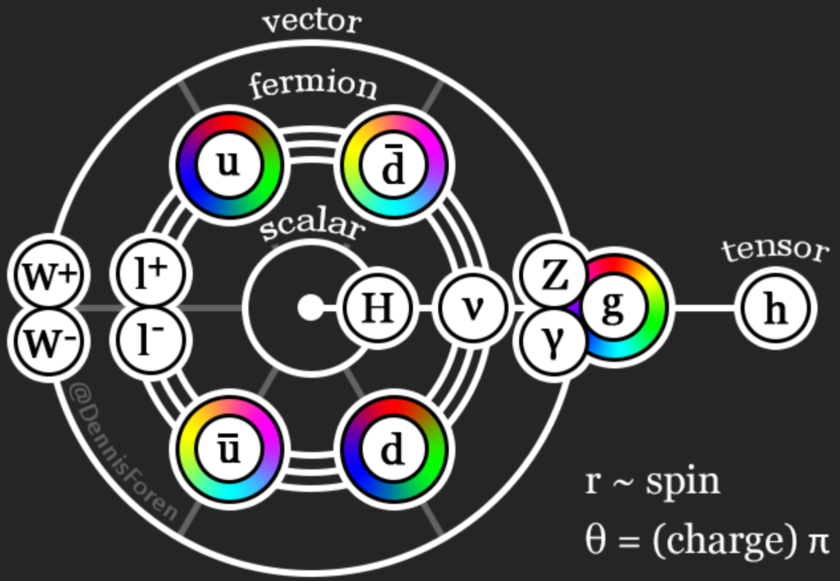
- Hierarchy Problem -



The Standard Model + Gravity

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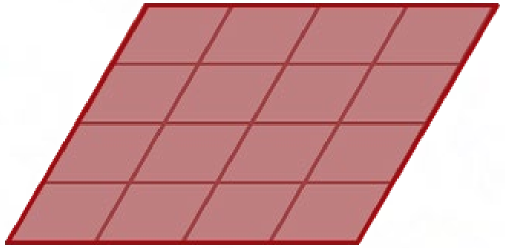

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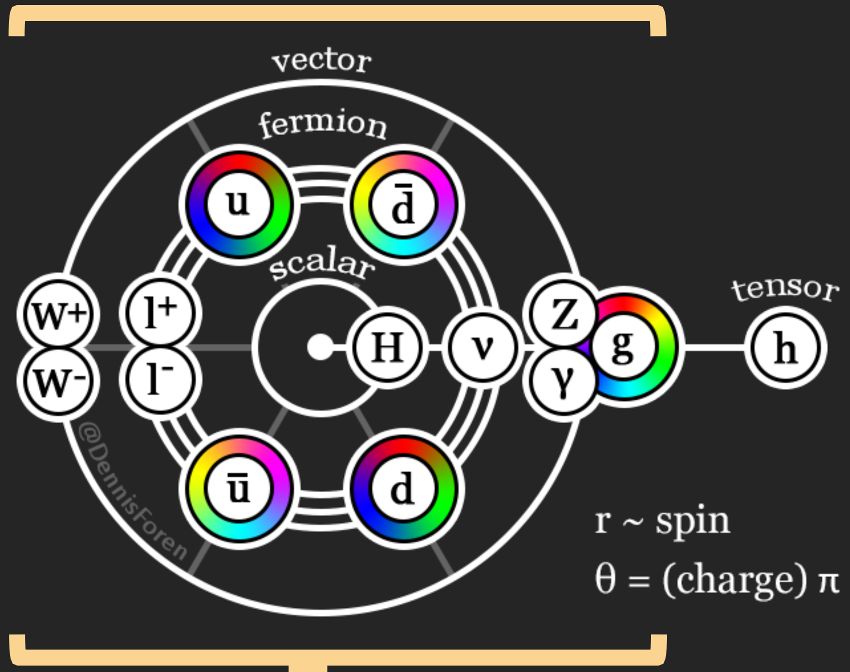
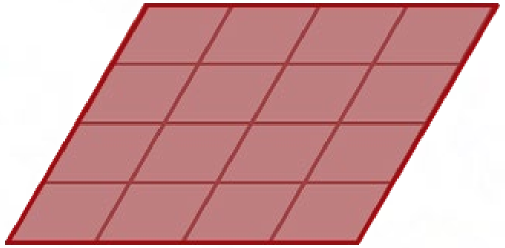


Electroweak

$$v_{\mathrm{EW}} = 0.246 \text{ TeV}$$

The Standard Model + Gravity

- Usual 4D Spacetime -



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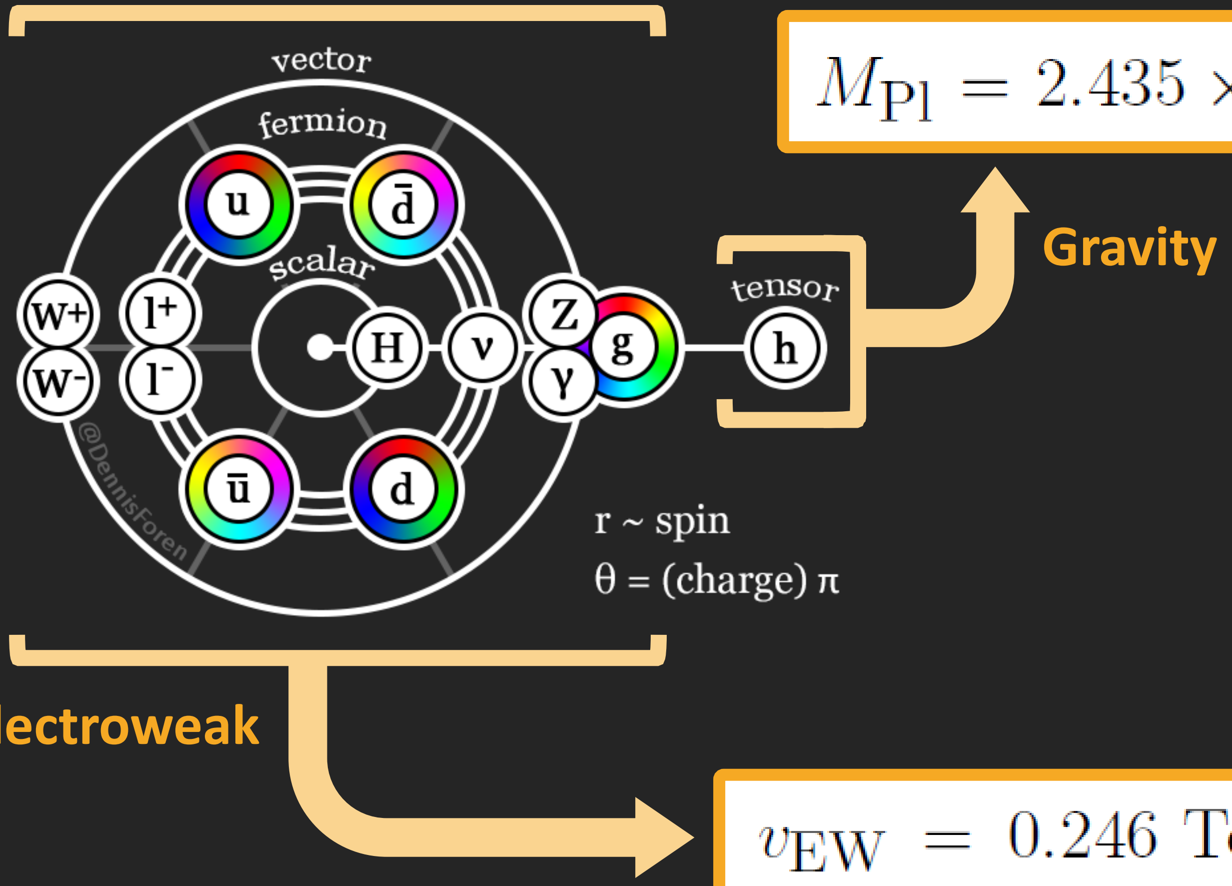
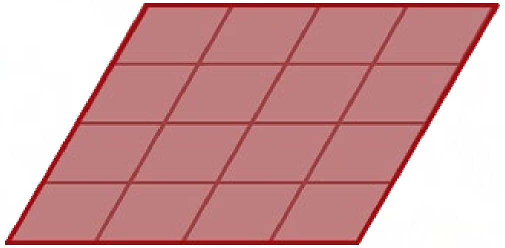


$$M_{\rm Pl} = 2.435 \times 10^{15} \text{ TeV}$$

$$v_{\rm EW} = 0.246 \text{ TeV}$$

The Standard Model + Gravity

- Usual 4D Spacetime -



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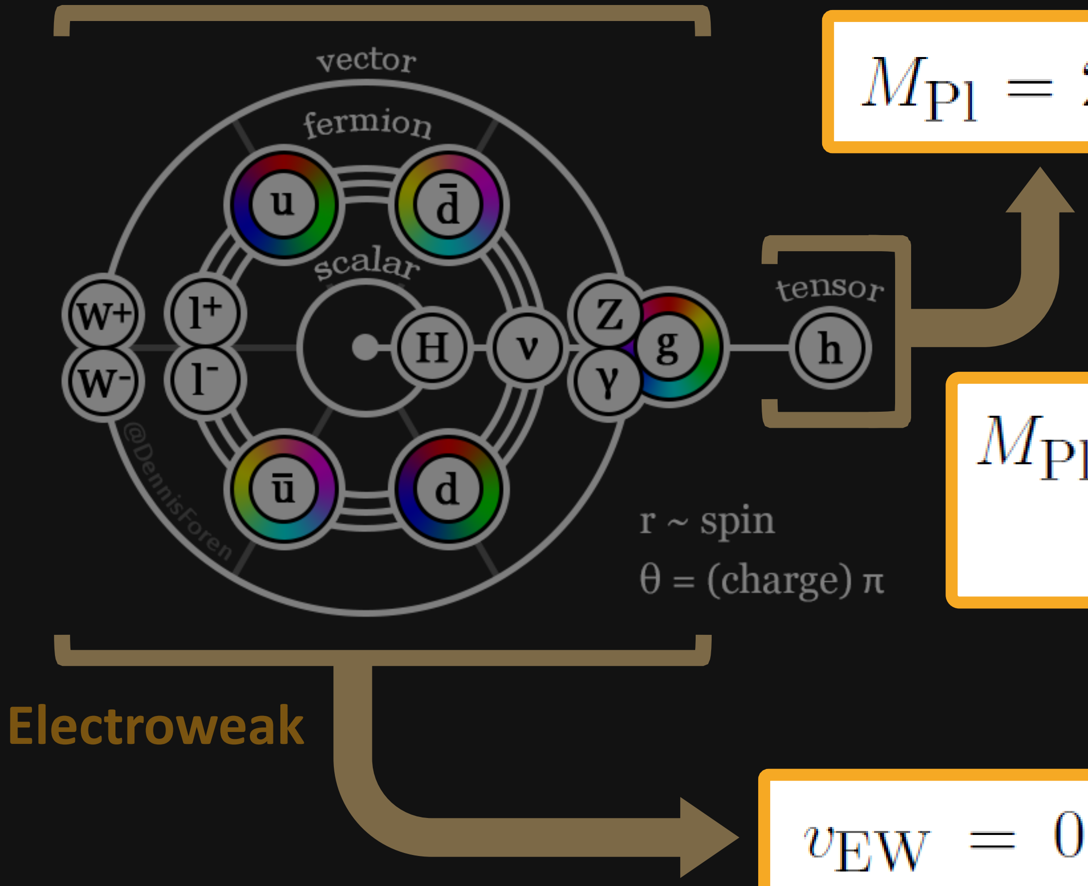


$$M_{\rm Pl} = 2.435 \times 10^{15}\ {\rm TeV}$$

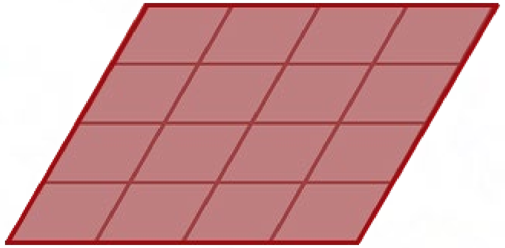
$$M_{\rm Pl}/v_{\rm EW} \sim 5 \times 10^{16}$$

How??

$$v_{\rm EW} = 0.246\ {\rm TeV}$$

The Randall-Sundrum 1 (RS1) Model

- Usual 4D Spacetime -



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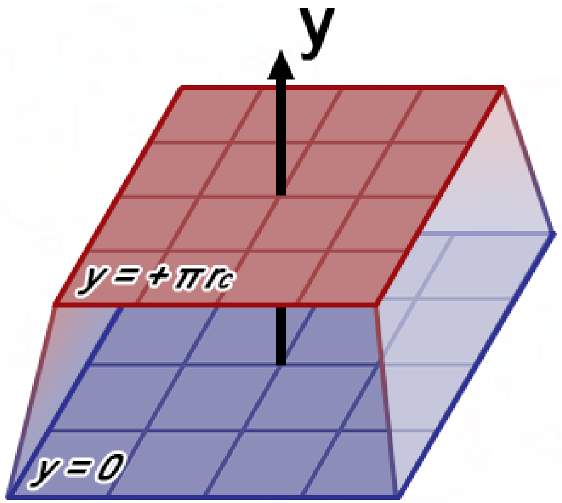
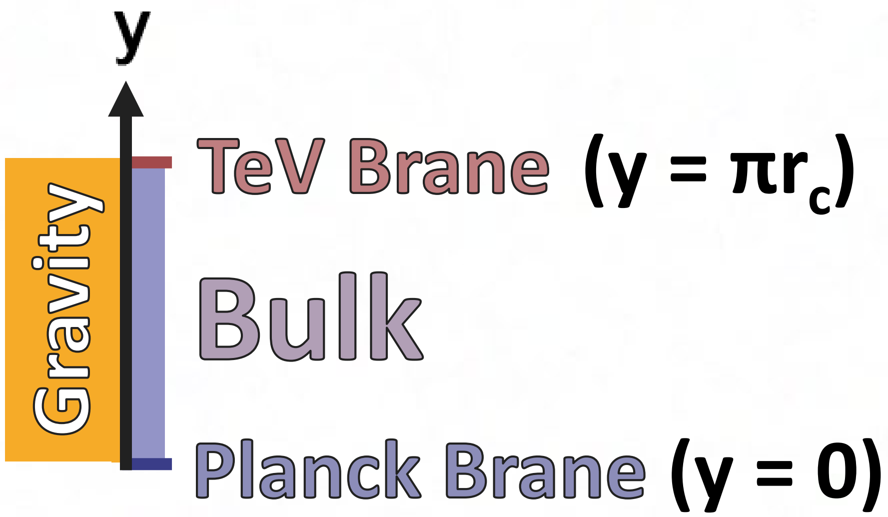
- 5D Randall-Sundrum 1 Spacetime -

A five-dimensional (5D) gravity theory with nonfactorizable geometry intro'd by Randall & Sundrum in 1999 to solve the hierarchy problem.

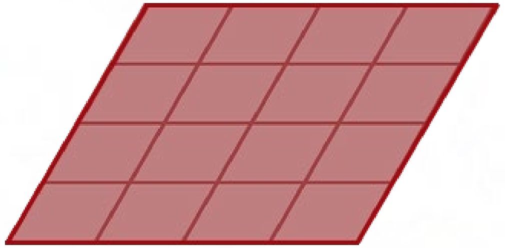


coordinates: $x^M = (x^\mu, y)$



The Randall-Sundrum 1 (RS1) Model

- Usual 4D Spacetime -



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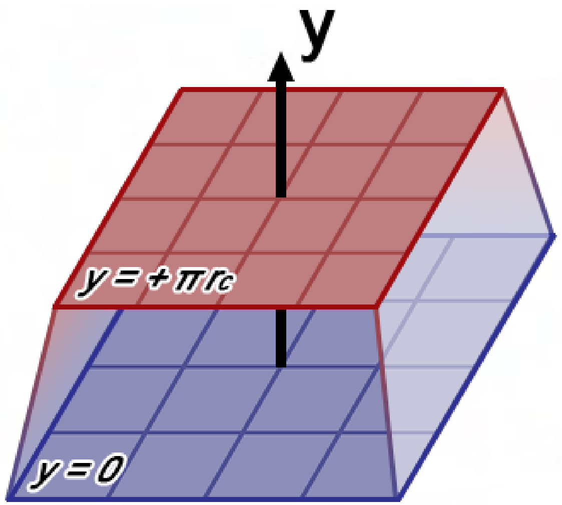

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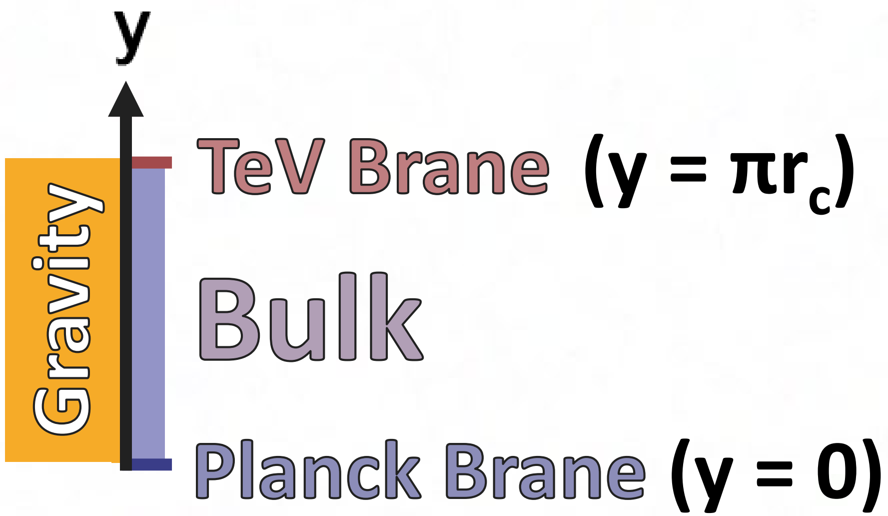

$M_{\rm Pl}$
@ y = 0

The Randall-Sundrum 1 (RS1) Model

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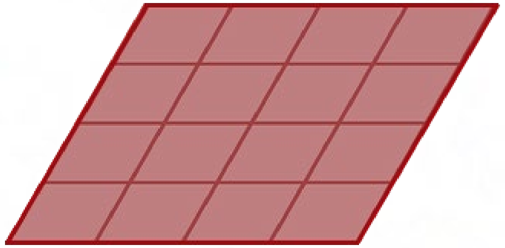

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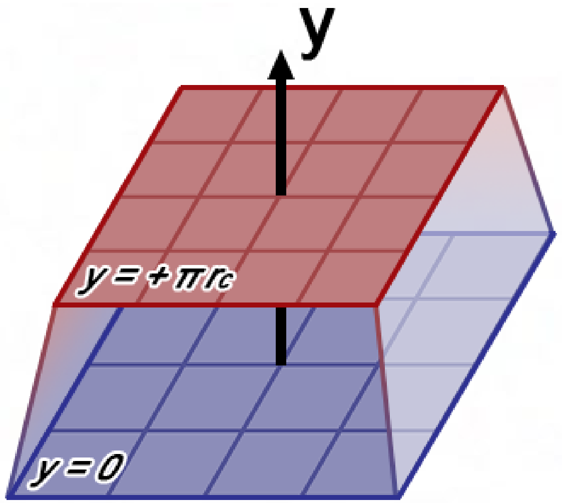

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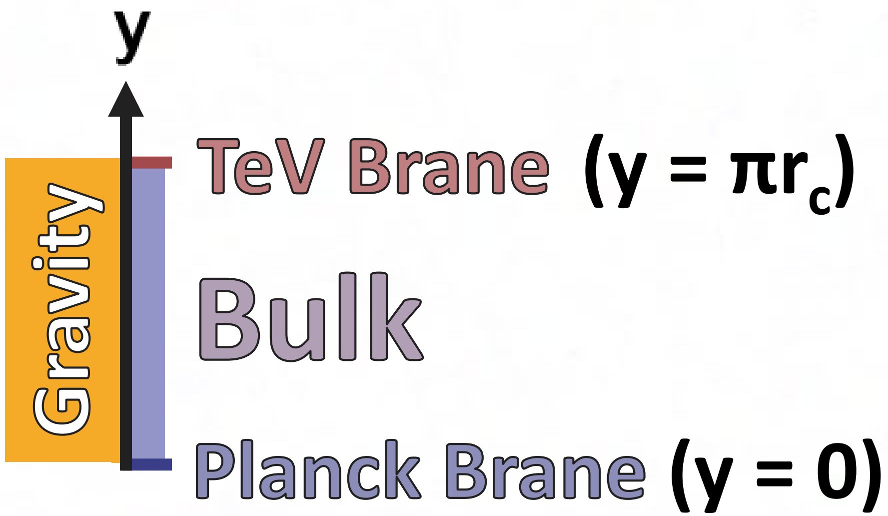
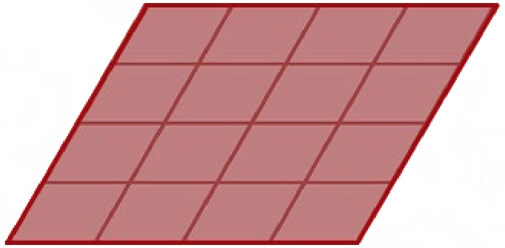


M_{Pl} @ y = 0 —warp→ $M_{\mathrm{Pl}}\, e^{-\pi k r_c}$ @ $y = \pi r_c$

The Randall-Sundrum 1 (RS1) Model

- Usual 4D Spacetime -



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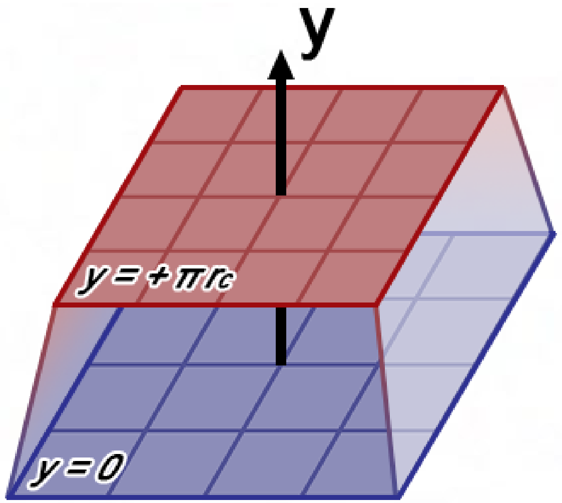

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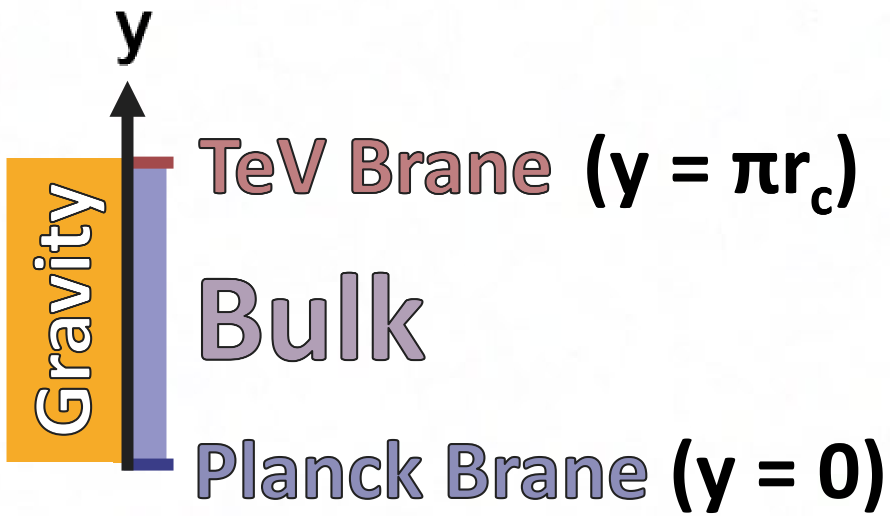
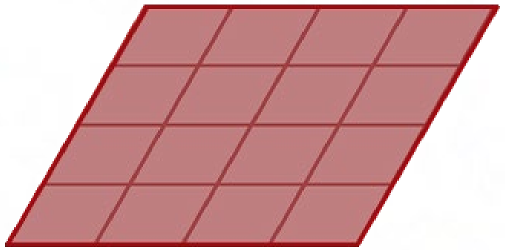


$$v_{\rm EW} \sim M_{\rm Pl}\, e^{-\pi k r_c} \qquad k r_c \sim 12$$

The Randall-Sundrum 1 (RS1) Model

- Usual 4D Spacetime -



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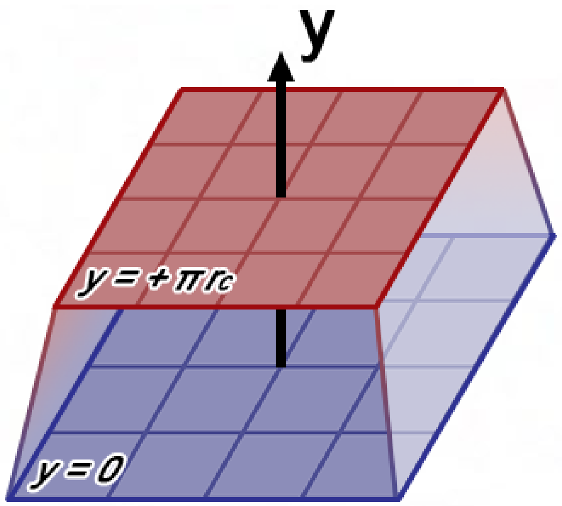

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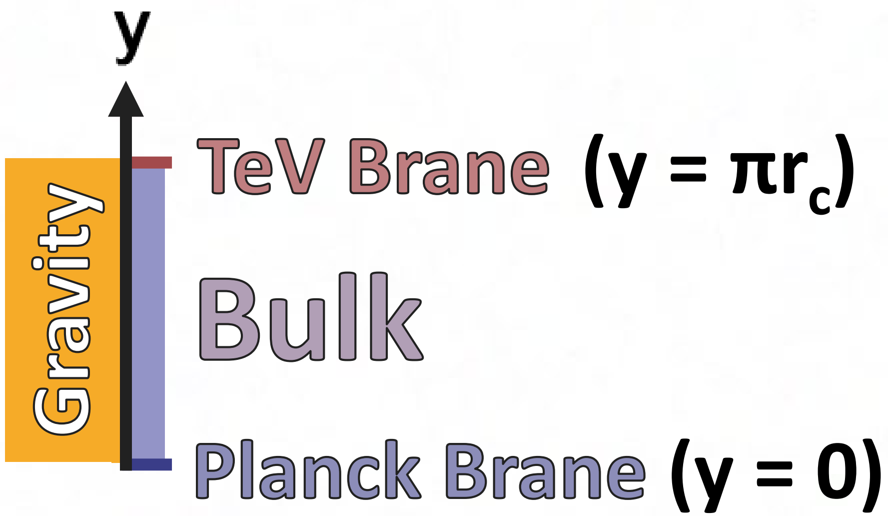
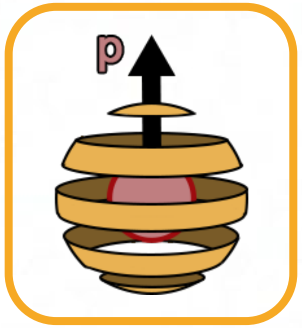


$$v_{\rm EW} \sim M_{\rm Pl}\, e^{-\pi k r_c} \qquad \sout{kr_c \sim 12}$$

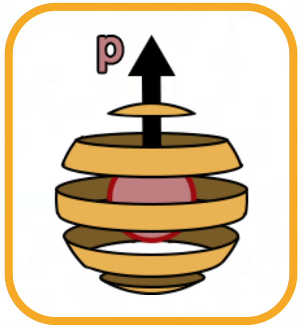
5D-to-4D: 4D Particle Content in RS1



5D RS1 Graviton

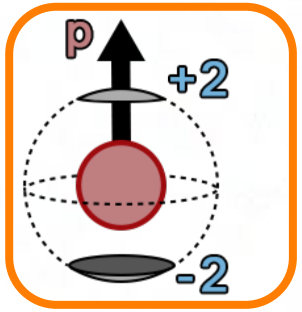
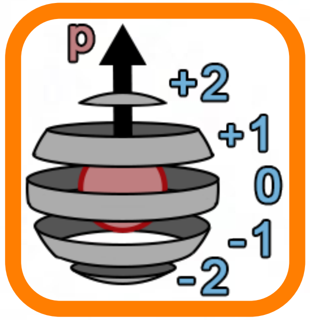
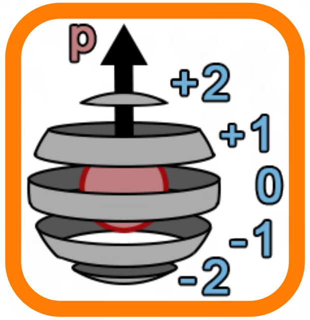
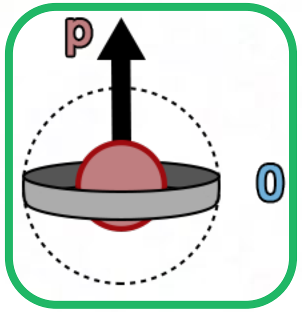
5D-to-4D: 4D Particle Content in RS1

Kaluza-Klein (KK) Decomposition



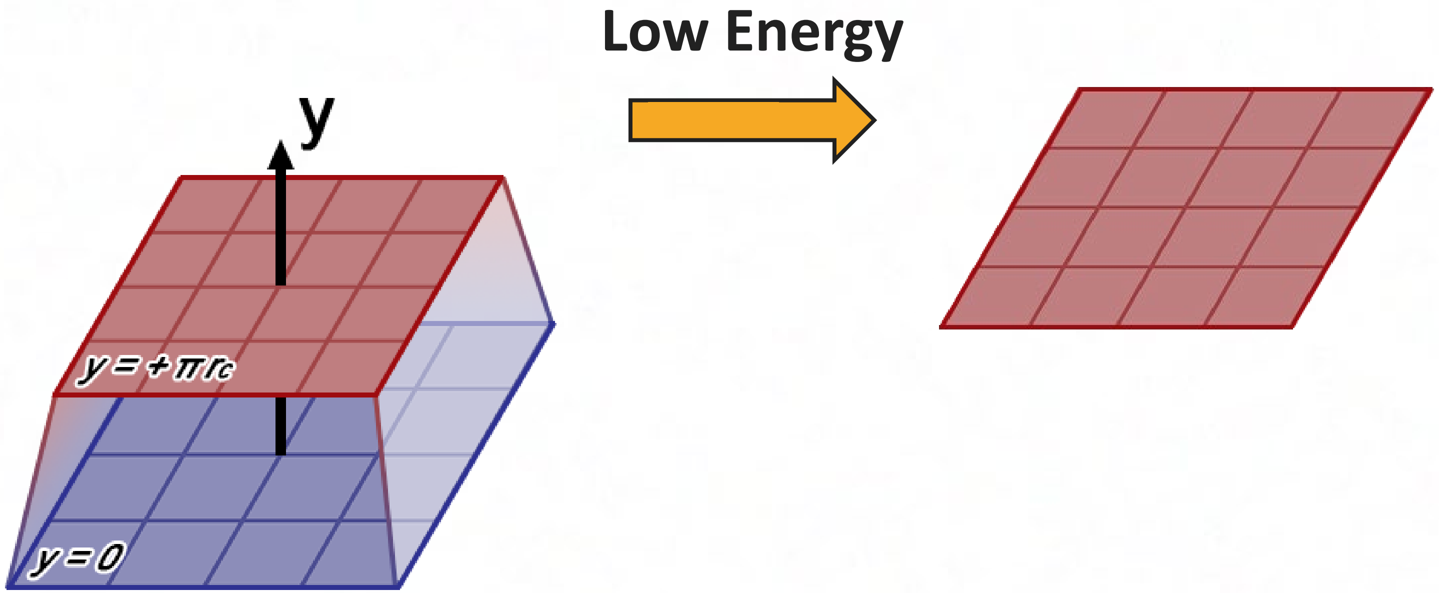
5D RS1 Graviton

$\supset$

KK Number:	$n = 0$	$n = 1$	$n = 2$	
$h_{\mu\nu}(x, y)$ Spin-2 KK Tower	 4D Graviton	& 	& 	& $\cdots$
$r(x)$ Spin-0 "KK Tower"	 Radion			

Massive Spin-2 States
(Massive KK modes)

Spontaneous Symmetry Breaking



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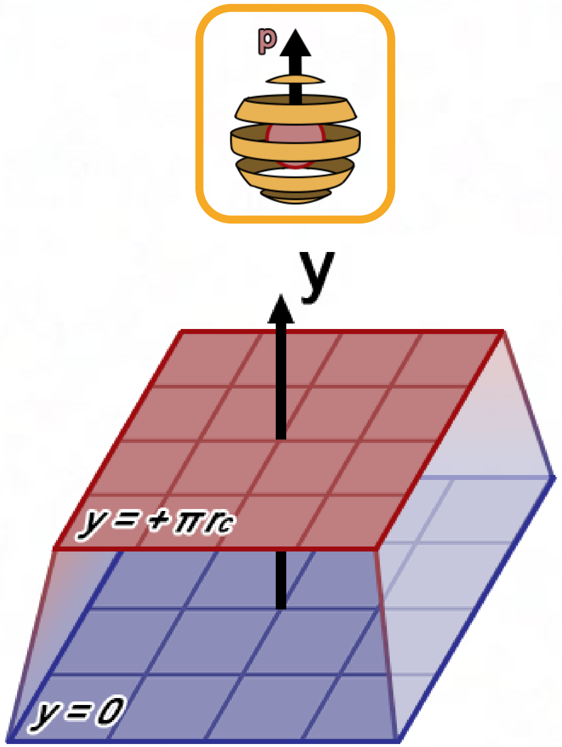
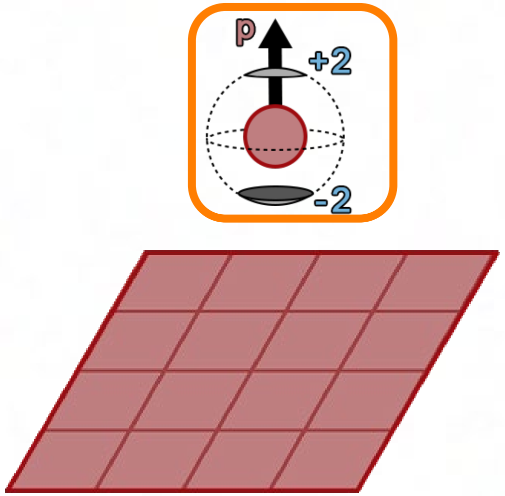


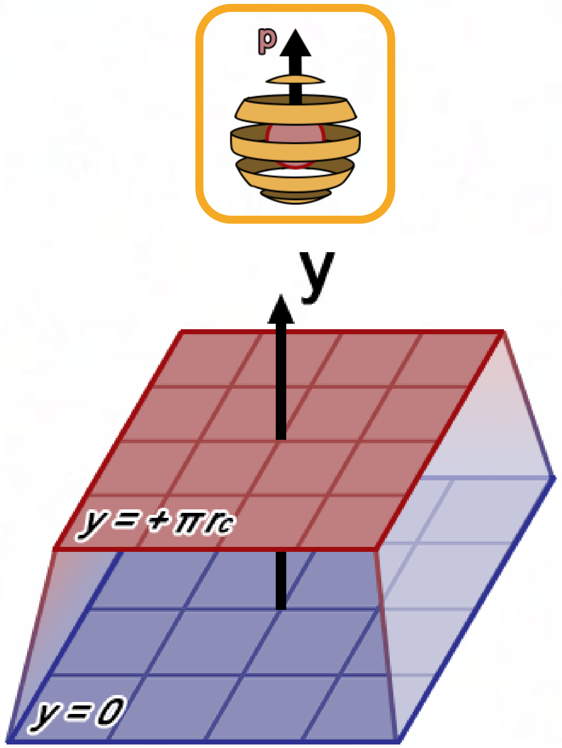
5D Diffeomorphism
Invariance

Low Energy



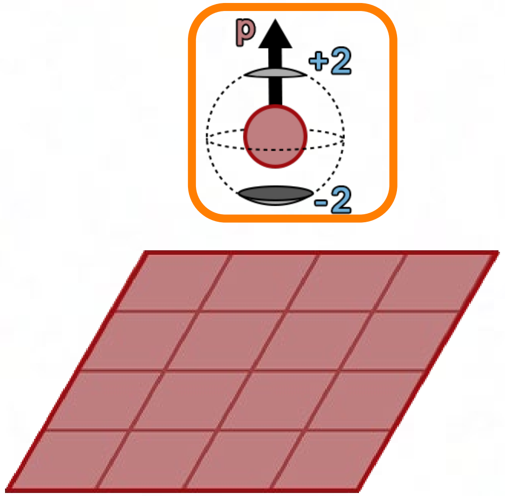
4D Diffeomorphism
Invariance

Spontaneous Symmetry Breaking



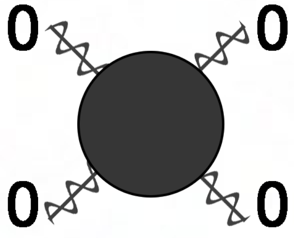
5D Diffeomorphism Invariance

Low Energy

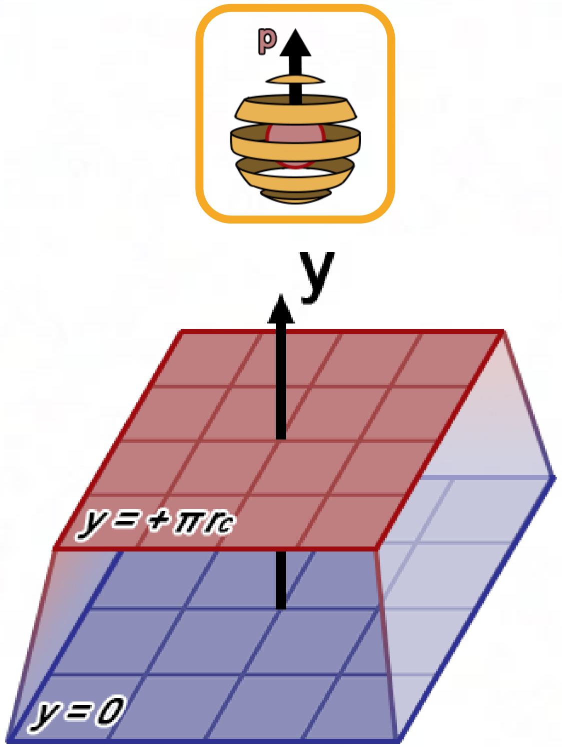


4D Diffeomorphism Invariance

4D Graviton Scattering

$\mathcal{M} =$  $\sim \frac{E^2}{M_{\rm Pl}^2} \sim \frac{s}{M_{\rm Pl}^2}$

Spontaneous Symmetry Breaking



5D Diffeomorphism Invariance

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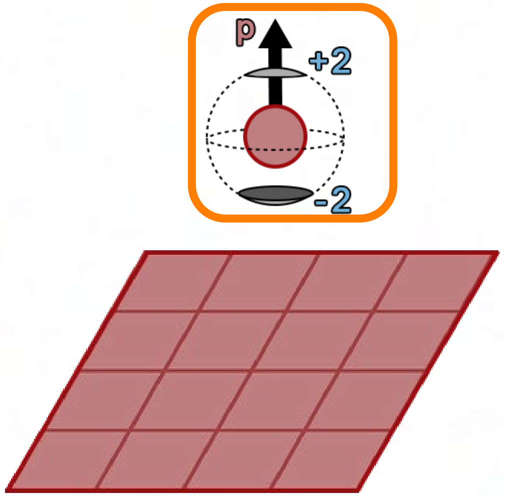

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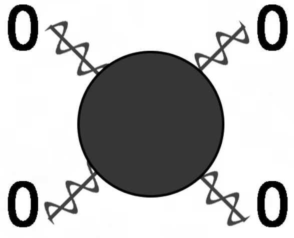
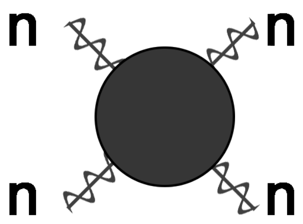


$\sim \frac{E^2}{M_{\mathrm{Pl}}^2} \sim \frac{s}{M_{\mathrm{Pl}}^2}$

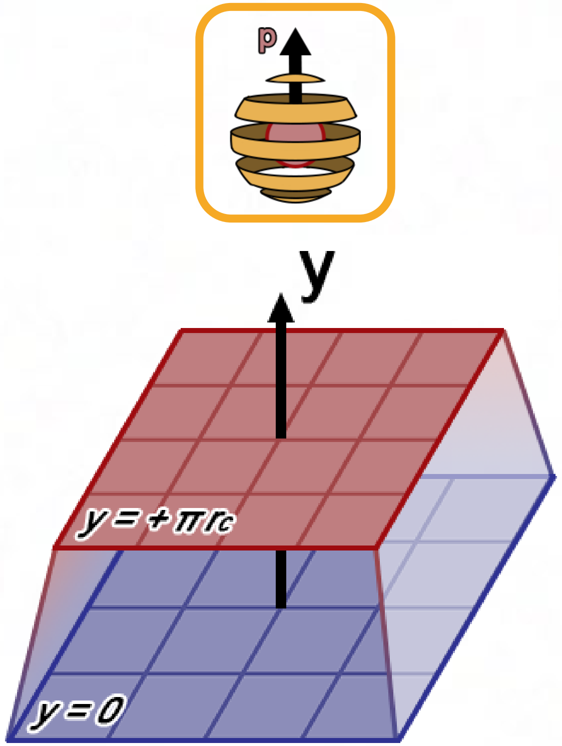
Massive KK Mode Scattering

$\mathcal{M} =$



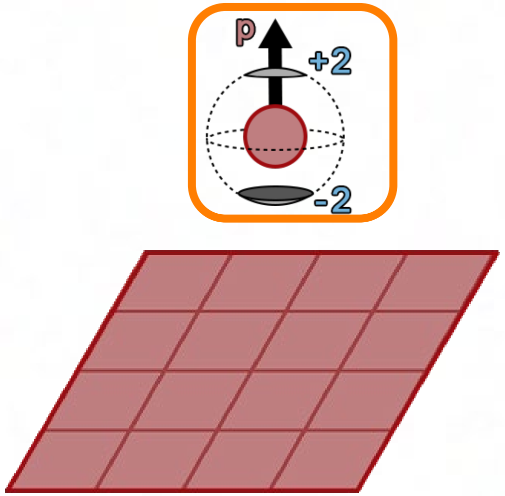
$\sim \mathcal{O}(s^5)$

Spontaneous Symmetry Breaking



5D Diffeomorphism Invariance

Low Energy



4D Diffeomorphism Invariance

4D Graviton Scattering

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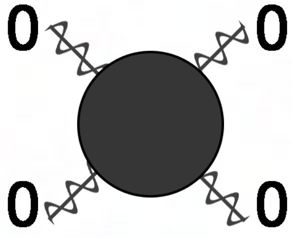

$\sim \frac{E^2}{M_{\mathrm{Pl}}^2} \sim \frac{s}{M_{\mathrm{Pl}}^2}$

Massive KK Mode Scattering

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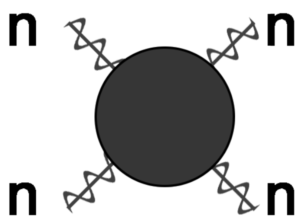
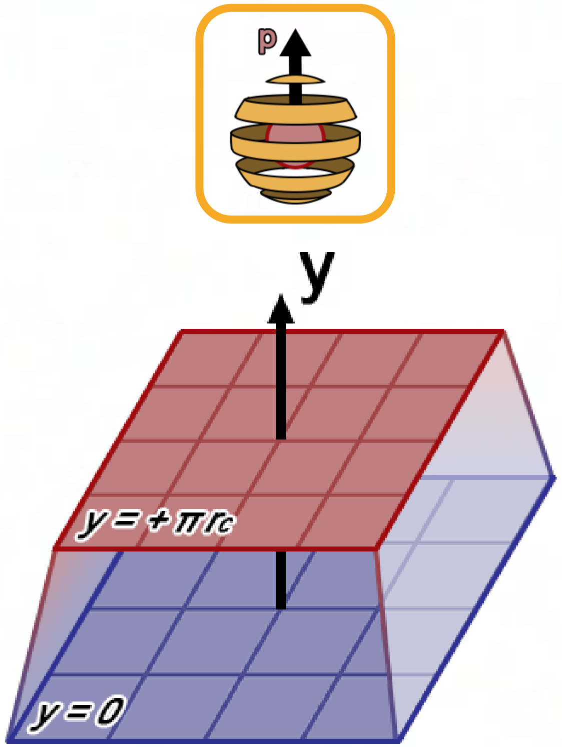


$\sim \mathcal{O}(s^5)$ → (Cancellations) $\sim \mathcal{O}(s)$

Spontaneous Symmetry Breaking



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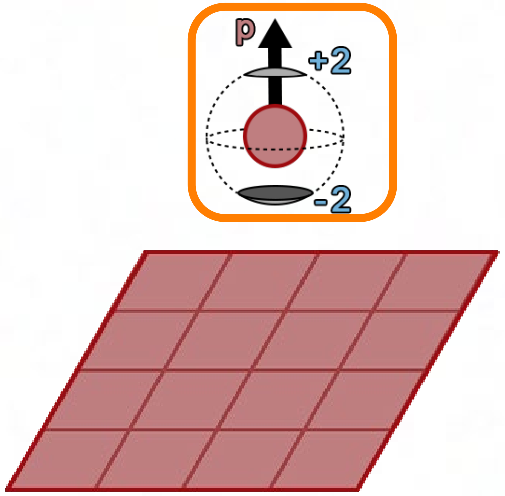
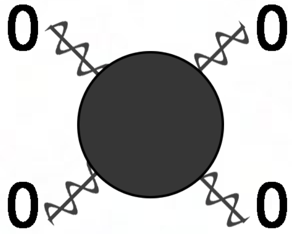
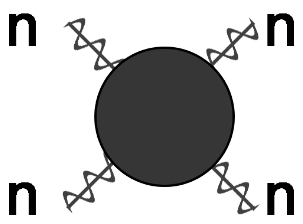


4D Diffeomorphism Invariance

4D Graviton Scattering

$\mathcal{M} =$  $\sim \frac{E^2}{M_{\mathrm{Pl}}^2} \sim \frac{s}{M_{\mathrm{Pl}}^2}$

Massive KK Mode Scattering

$\mathcal{M} =$  $\sim \mathcal{O}(s^5)$ → $\sim \mathcal{O}(s)$

Cancellations

- New Parameterization of RS1 Model -

- Explicit parameterization of 5D RS1 Lagrangian.
- New term to eliminate all “cosmological constant”-like terms to all orders.
- A **5D-to-4D formula** to obtain 4D Effective RS1 Lagrangian.

- Massive Spin-2 KK Mode Scattering -

- Demonstrated **O(s) growth** in tree-level 2-to-2 massive spin-2 KK mode scattering in RS1 and the 5DOT.
- Derivation & proofs of **sum rules** (recently generalized to inelastic).
- Simultaneously confirmed numerically.

- Analysis of the 4D Effective RS1 Model -

- Confirmation of 5D **strong-coupling scale** in the 4D effective model.
- Analysis of **KK tower truncation** on the 4D effective model accuracy.

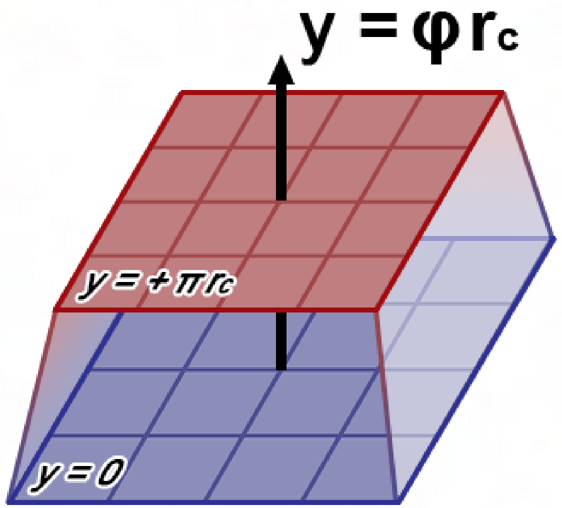
(when presenting my original results, I will label w/ corresponding papers info)

From 5D RS1 Lagrangian to 4D Particle Interactions

5D RS1 Metric

$$ds^2 = G_{\mu\nu}\, dx^\mu\, dx^\nu + G_{55}\, dy\, dy$$

$$G_{MN} = \begin{pmatrix} w(x,y)\, g_{\mu\nu} & 0 \\ 0 & -v(x,y)^2 \end{pmatrix}$$



- Spin-2 Tower -

$$g_{\mu\nu}(x,y) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x,y)$$

- Radion -

$$w(x,y) \equiv e^{-2k|y|} e^{-2u(x)}$$

$$v(x,y) \equiv 1 + 2u(x)$$

$$u(x,y) \equiv \frac{\kappa \hat{r}(x)}{2\sqrt{6}} e^{+k(2|y| - \pi r_c)}$$

- 5D RS1 Lagrangian -

$$\mathcal{L}_{\rm 5D} = \mathcal{L}_{\rm EH} + \mathcal{L}_{\rm CC} + \Delta\mathcal{L}$$

- 5D RS1 Lagrangian -

$$\mathcal{L}_{\text{5D}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{CC}} + \Delta\mathcal{L}$$

- Einstein-Hilbert Lagrangian -

$$\mathcal{L}_{\text{EH}} \equiv -\frac{2}{\kappa^2}\sqrt{G}\,R \qquad \cong -\frac{2}{\kappa^2}\sqrt{G}\,\tilde{G}^{MN}\left[\Gamma^{Q}_{MP}\Gamma^{P}_{NQ} - \Gamma^{Q}_{MN}\Gamma^{P}_{PQ}\right]$$

... implies discontinuous curvature at branes.

$$\Gamma^{P}_{MN} \equiv \frac{1}{2}\tilde{G}^{PQ}(\partial_M G_{NQ} + \partial_N G_{MQ} - \partial_Q G_{MN})$$

$$G_{MN} = \begin{pmatrix} w(x,y)\,g_{\mu\nu} & 0 \\ 0 & -v(x,y)^2 \end{pmatrix}$$

- 5D RS1 Lagrangian -

$$\mathcal{L}_{\text{5D}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{CC}} + \Delta\mathcal{L}$$

- Cosmological Constant Lagrangian-

$$\mathcal{L}_{\text{CC}} \equiv -\frac{2}{\kappa^2}\left[-12k^2\sqrt{G} + 6k\sqrt{\overline{G}}\,(\partial_y^2|y|)\right]$$

(bulk CC) + (brane tensions)

$$(\partial_y^2|y|) = 2\left[\delta(y) - \delta(y - \pi r_c)\right]$$

$$G_{MN} = \begin{pmatrix} w(x,y)\, g_{\mu\nu} & 0 \\ 0 & -v(x,y)^2 \end{pmatrix} \qquad \overline{G}_{MN} = \begin{pmatrix} w(x,y)\, g_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix}$$

- 5D RS1 Lagrangian -

$$\mathcal{L}_{\text{5D}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{CC}} + \Delta\mathcal{L}$$

- Total Derivative -

[Dissertation]
[2002.12458]

$$\Delta\mathcal{L} \equiv \frac{12k}{\kappa^2}\partial_y \left[w^2 \sqrt{-g}\,(\partial_y |y|) \right]$$

Ensures…
- CC-like terms cancel
- 2 derivatives per term
- Derivatives act on (different) fields

- A-Type Terms -

2 4D derivatives, e.g.

$$\partial_\mu \hat{h}(x,y)\,\partial_\nu \hat{h}(x,y)\,\hat{h}(x,y)$$

$\mathcal{O}(E^{+2})$

- B-Type Terms -

2 extra-dim. derivatives, e.g.

$$\partial_y \hat{h}(x,y)\,\partial_y \hat{h}(x,y)\,\hat{h}(x,y)$$

$\mathcal{O}(E^{0})$

5D RS1 Lagrangian: Weak Field Expansion

$$g_{\mu\nu}(x,y) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x,y)$$

$$u(x,y) \equiv \frac{\kappa \hat{r}(x)}{2\sqrt{6}} e^{+k(2|y|-\pi r_c)}$$

$$\mathcal{L}_{5\text{D}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{CC}} + \Delta\mathcal{L}$$

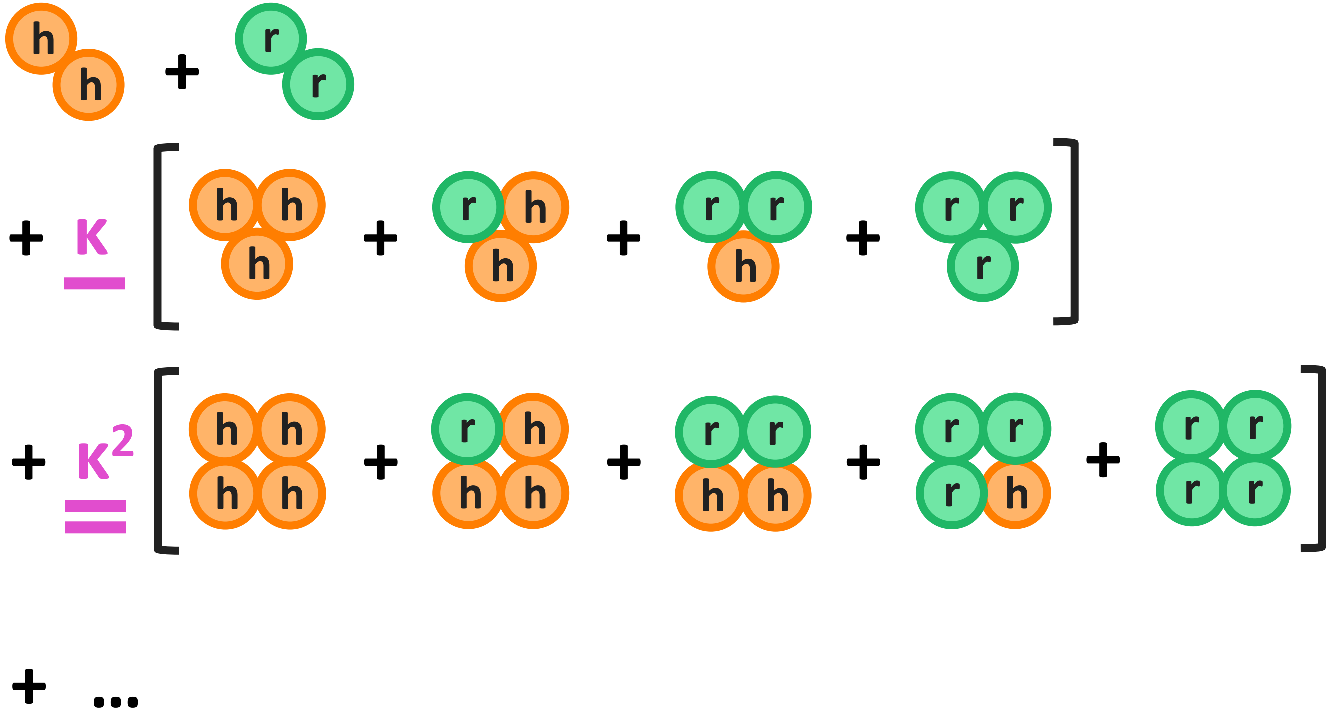
5D RS1 Lagrangian: Weak Field Expansion

$$g_{\mu\nu}(x,y) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x,y)$$

$$u(x,y) \equiv \frac{\kappa \hat{r}(x)}{2\sqrt{6}} e^{+k(2|y|-\pi r_c)}$$

- Weak Field Expanded (WFE) 5D RS1 Lagrangian -

$$\mathcal{L}_{5\text{D}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{CC}} + \Delta\mathcal{L} =$$

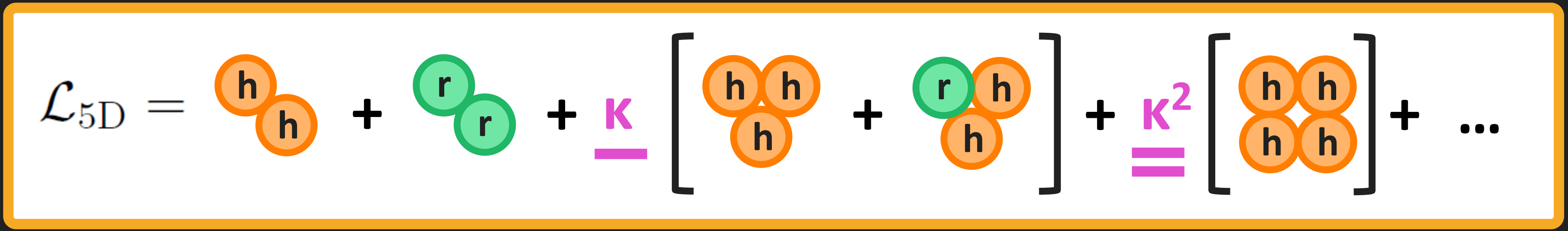


[Dissertation]

[2002.12458]

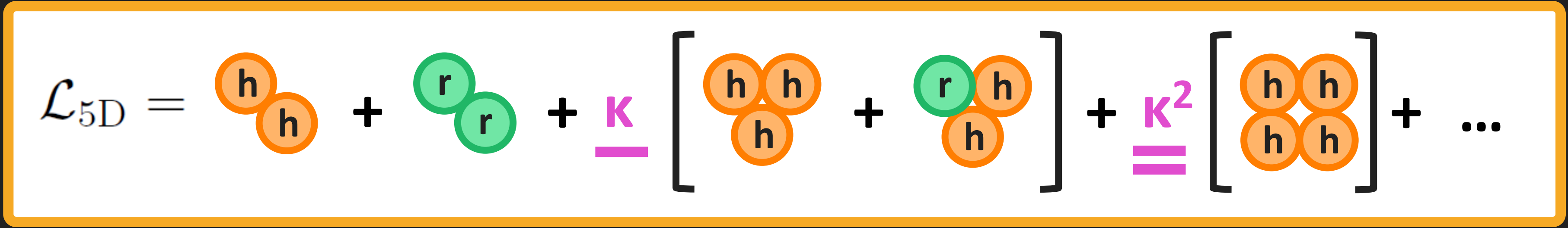
- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian



- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian

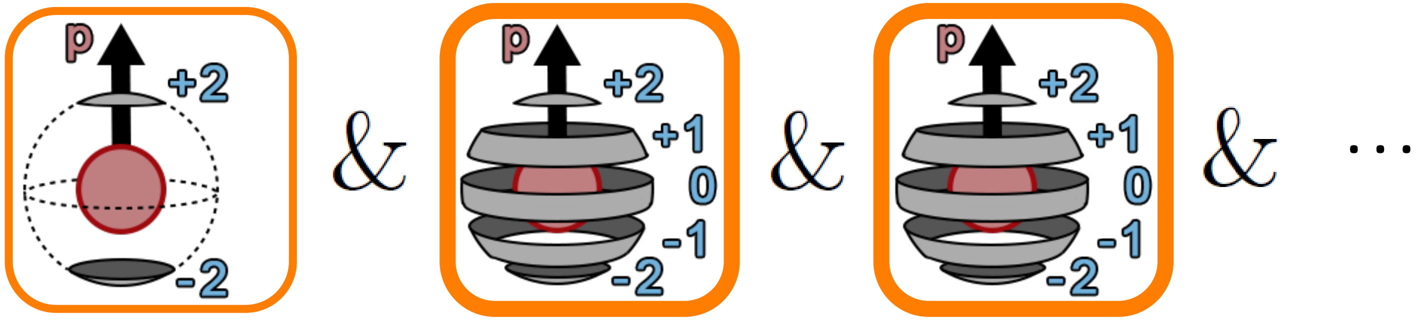


Kaluza-Klein (KK) Tower

KK Number: $n = 0$ | $n = 1$ | $n = 2$

h $h_{\mu\nu}(x, y)$

Spin-2 Tower

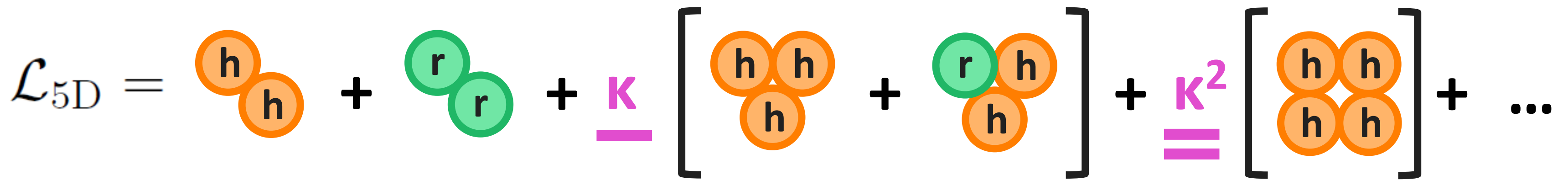


4D Graviton

Massive Spin-2 States
(Massive KK modes)

- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian

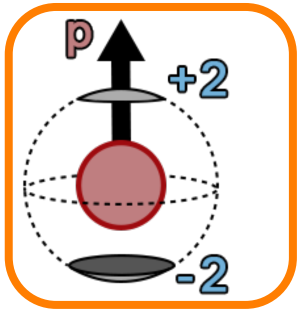


Kaluza-Klein (KK) Tower

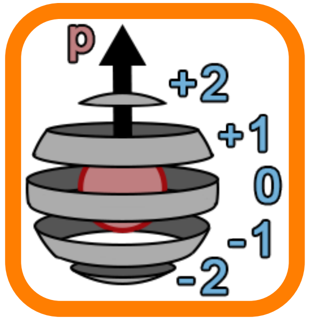
KK Number:	$n = 0$	$n = 1$	$n = 2$

h $h_{\mu\nu}(x, y)$

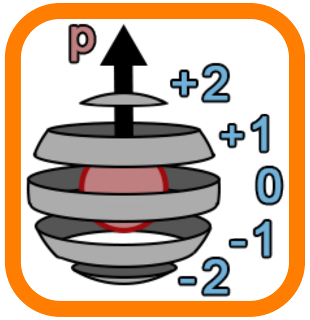
Spin-2 Tower



&



&



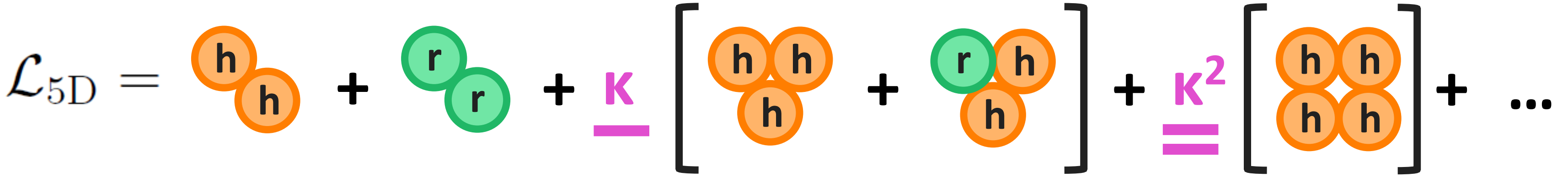
& $\cdots$

$$\hat{h}_{\mu\nu}(x, y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(y)$$

KK Decomposition

- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian



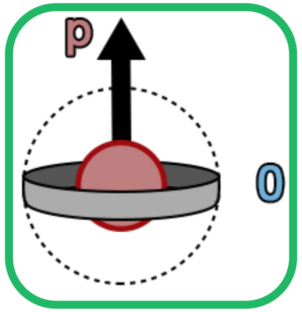
Kaluza-Klein (KK) Tower

KK Number:	$n = 0$	$n = 1$	$n = 2$

r

$r(x)$

Spin-0 "Tower"



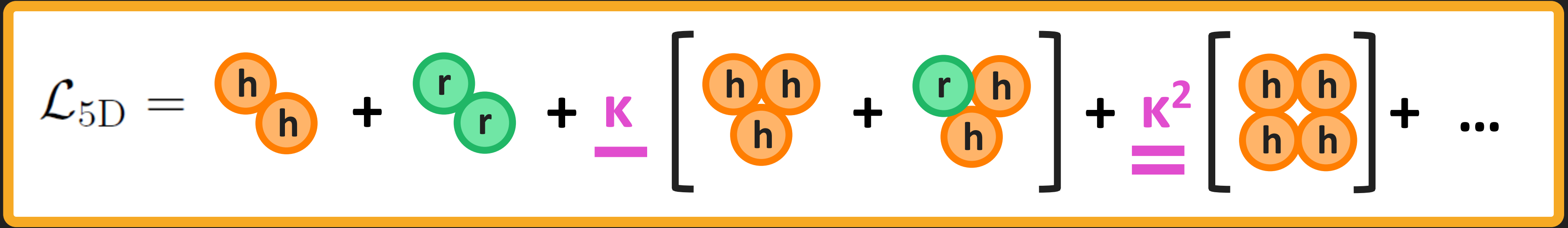
Radion

$$\hat{r}_{\mu\nu}(x) = \frac{1}{\sqrt{\pi r_c}}\,\hat{r}^{(0)}(x)\,\psi_0$$

KK Decomposition

- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian



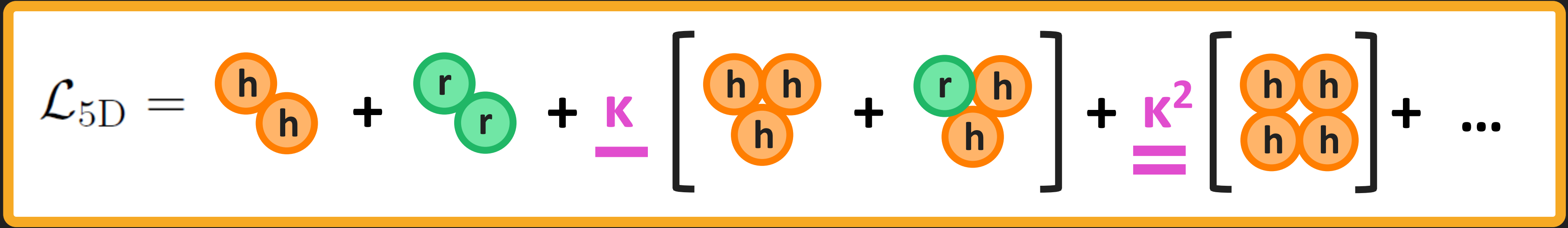
KK decomp.

$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(y)$$

$$\hat{r}_{\mu\nu}(x) = \frac{1}{\sqrt{\pi r_c}} \hat{r}^{(0)}(x)\, \psi_0$$

- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian



KK decomp.

$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(y)$$

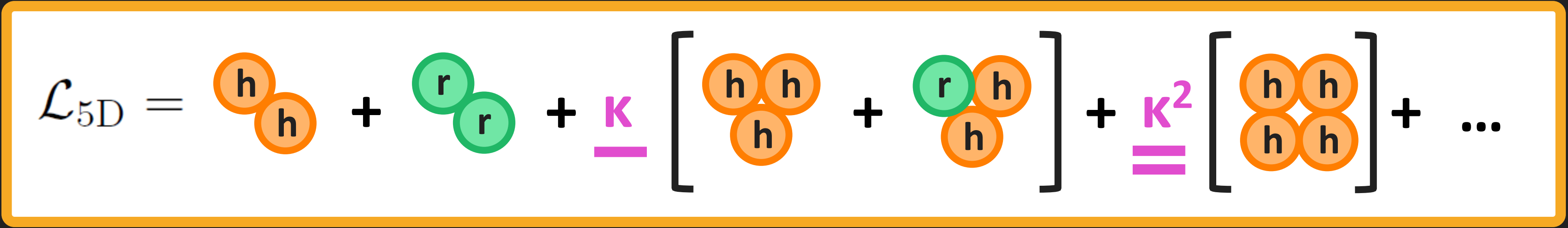
$$\hat{r}_{\mu\nu}(x) = \frac{1}{\sqrt{\pi r_c}} \hat{r}^{(0)}(x)\, \psi_0$$

Integrate extra dim.

$$\mathcal{L}^{(\text{eff})}_{4D} = \int_{-\pi r_c}^{+\pi r_c} dy \quad \mathcal{L}_{5D}$$

- WFE 5D RS1 Lagrangian -

5D WFE RS1 Lagrangian



KK decomp.

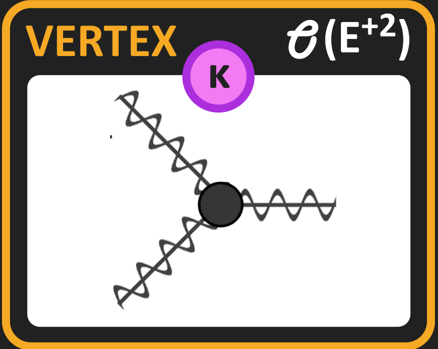
$$\hat{h}_{\mu\nu}(x,y) = \frac{1}{\sqrt{\pi r_c}} \sum_{n=0}^{+\infty} \hat{h}^{(n)}_{\mu\nu}(x)\, \psi_n(y)$$

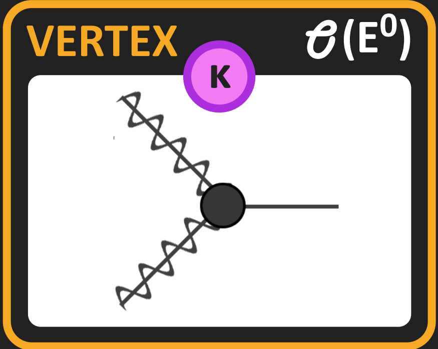
$$\hat{r}_{\mu\nu}(x) = \frac{1}{\sqrt{\pi r_c}} \hat{r}^{(0)}(x)\, \psi_0$$

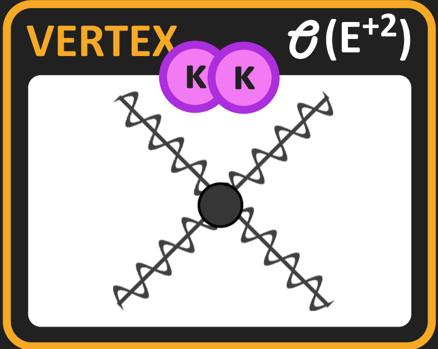
Integrate extra dim.

$$\mathcal{L}^{(\text{eff})}_{4\text{D}} = \int_{-\pi r_c}^{+\pi r_c} dy \;\; \mathcal{L}_{5\text{D}}$$

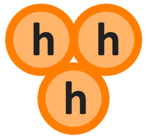
Interactions b/w 4D states







5D-to-4D: Example



$$\hat{h}(x,y)\,\hat{h}(x,y)\,\hat{h}(x,y)$$

KK Decompose

$$\left[\frac{1}{\sqrt{\pi r_c}}\sum_{l=0}^{+\infty}\hat{h}^{(l)}(x)\,\psi_l(y)\right]\left[\frac{1}{\sqrt{\pi r_c}}\sum_{m=0}^{+\infty}\hat{h}^{(m)}(x)\,\psi_m(y)\right]\left[\frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}^{(n)}(x)\,\psi_n(y)\right]$$

5D-to-4D: Example

$$\hat{h}(x,y)\,\hat{h}(x,y)\,\hat{h}(x,y)$$

KK Decompose

$$\left[\frac{1}{\sqrt{\pi r_c}}\sum_{l=0}^{+\infty}\hat{h}^{(l)}(x)\,\psi_l(y)\right]\left[\frac{1}{\sqrt{\pi r_c}}\sum_{m=0}^{+\infty}\hat{h}^{(m)}(x)\,\psi_m(y)\right]\left[\frac{1}{\sqrt{\pi r_c}}\sum_{n=0}^{+\infty}\hat{h}^{(n)}(x)\,\psi_n(y)\right]$$

$$\frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

4D coordinates **extra-dim.**

5D-to-4D: Example

h h h

$$\hat{h}(x,y)\,\hat{h}(x,y)\,\hat{h}(x,y) = \frac{1}{(\sqrt{\pi r_c})^3} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right] \left[\psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

4D coordinates

extra-dim.

$$\mathcal{L}_{\mathrm{4D}}^{(\mathrm{eff})} = \int_{-\pi r_c}^{+\pi r_c} dy \quad \mathcal{L}_{\mathrm{5D}}$$

5D-to-4D: Example

h h h

$$\hat{h}(x,y)\,\hat{h}(x,y)\,\hat{h}(x,y) = \frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

4D coordinates **extra-dim.**

$$\mathcal{L}_{4D}^{(\text{eff})} = \int_{-\pi r_c}^{+\pi r_c} dy \quad \mathcal{L}_{5D}$$

$$\frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\int_{-\pi r_c}^{+\pi r_c} dy \quad \psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

coupling integral

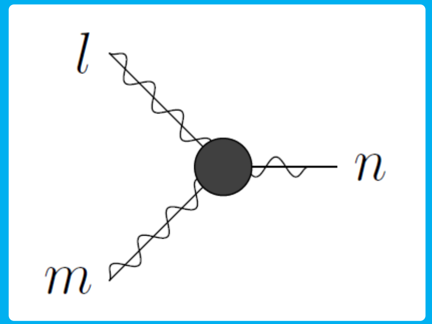
5D-to-4D: Example

h h h

$$\hat{h}(x,y)\,\hat{h}(x,y)\,\hat{h}(x,y) = \frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

4D coordinates — **extra-dim.**

$$\mathcal{L}_{4D}^{(\text{eff})} = \int_{-\pi r_c}^{+\pi r_c} dy \quad \mathcal{L}_{5D}$$



Cubic interaction b/w spin-2 modes

$$\frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\hat{h}^{(l)}(x)\,\hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\int_{-\pi r_c}^{+\pi r_c} dy \quad \psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

coupling integral

A-Type = Two Regular 4D Derivatives

h h h

two 4D derivatives	$\underbrace{\partial_\mu \hat{h}(x,y)}\,\underbrace{\partial_\nu \hat{h}(x,y)}\,\hat{h}(x,y)$	$\mathcal{O}(E^{+2})$

A-Type = Two Regular 4D Derivatives

h h h

two 4D derivatives

$$\partial_\mu \hat{h}(x,y)\, \partial_\nu \hat{h}(x,y)\, \hat{h}(x,y)$$

$\mathcal{O}$ (E^{+2})

$$\frac{1}{(\sqrt{\pi r_c})^3} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \left[\partial_\mu \hat{h}^{(l)}(x)\, \partial_\nu \hat{h}^{(m)}(x)\, \hat{h}^{(n)}(x)\right] \left[\int_{-\pi r_c}^{+\pi r_c} dy \quad e^{-2k|y|}\, \psi_l(y)\, \psi_m(y)\, \psi_n(y)\right]$$

A-Type = Two Regular 4D Derivatives

two 4D derivatives

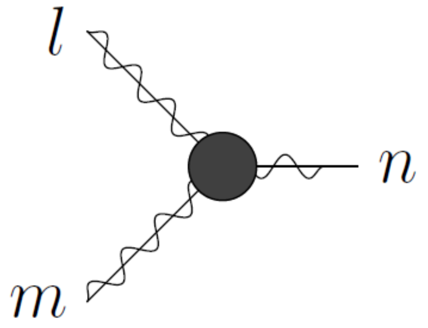
$$\partial_\mu \hat{h}(x,y)\,\partial_\nu \hat{h}(x,y)\,\hat{h}(x,y)$$

$\mathcal{O}(E^{+2})$

$$\frac{1}{(\sqrt{\pi r_c})^3}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\partial_\mu \hat{h}^{(l)}(x)\,\partial_\nu \hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]\left[\int_{-\pi r_c}^{+\pi r_c} dy \quad e^{-2k|y|}\,\psi_l(y)\,\psi_m(y)\,\psi_n(y)\right]$$

$$\frac{1}{\sqrt{\pi r_c}}\sum_{l=0}^{+\infty}\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\left[\partial_\mu \hat{h}^{(l)}(x)\,\partial_\nu \hat{h}^{(m)}(x)\,\hat{h}^{(n)}(x)\right]a_{lmn}$$

A-type coupling



B-Type = Two Extra-Dimensional Derivatives

h h h

two extra-dim. derivatives $\partial_y \hat{h}(x,y)\, \partial_y \hat{h}(x,y)\, \hat{h}(x,y)$ $\mathcal{O}(E^0)$

B-Type = Two Extra-Dimensional Derivatives

two extra-dim. derivatives $\quad \partial_y \hat{h}(x,y)\, \partial_y \hat{h}(x,y)\, \hat{h}(x,y) \quad \mathcal{O}(E^0)$

$$\frac{1}{(\sqrt{\pi r_c})^3} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \left[\hat{h}^{(l)}(x)\, \hat{h}^{(m)}(x)\, \hat{h}^{(n)}(x)\right] \left[\int_{-\pi r_c}^{+\pi r_c} dy \quad e^{-4k|y|}\, \partial_y \psi_l(y)\, \partial_y \psi_m(y)\, \psi_n(y)\right]$$

B-Type = Two Extra-Dimensional Derivatives

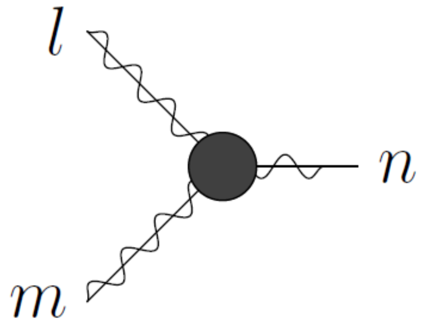
h h h

two extra-dim. derivatives $\partial_y \hat{h}(x,y)\, \partial_y \hat{h}(x,y)\, \hat{h}(x,y)$ $\mathcal{O}(E^0)$

$$\frac{1}{(\sqrt{\pi r_c})^3} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \left[\hat{h}^{(l)}(x)\, \hat{h}^{(m)}(x)\, \hat{h}^{(n)}(x) \right] \left[\int_{-\pi r_c}^{+\pi r_c} dy \quad e^{-4k|y|}\, \partial_y \psi_l(y)\, \partial_y \psi_m(y)\, \psi_n(y) \right]$$

$$\frac{1}{\sqrt{\pi r_c}} \sum_{l=0}^{+\infty} \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} \left[\hat{h}^{(l)}(x)\, \hat{h}^{(m)}(x)\, \hat{h}^{(n)}(x) \right] \frac{1}{r_c^2}\, b_{l'm'n}$$

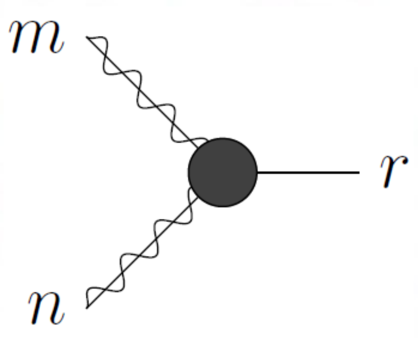
B-type coupling



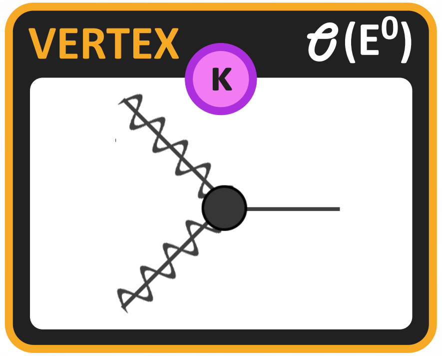
Relevant Interactions & Coupling Integrals

- Radion Coupling -

h h r

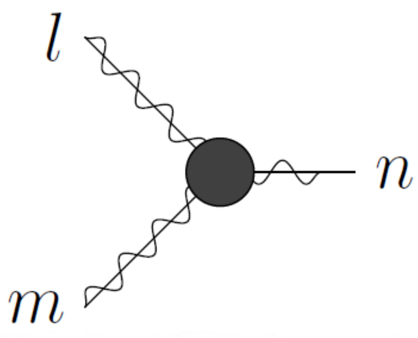


$b_{m'n'r}$

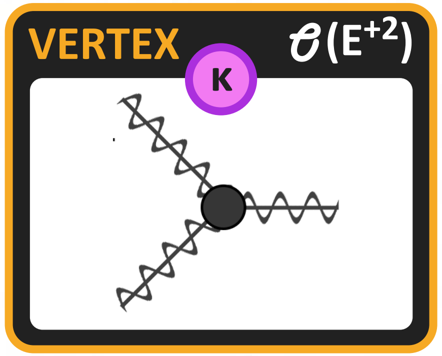


- Cubic Spin-2 Coupling -

h h h

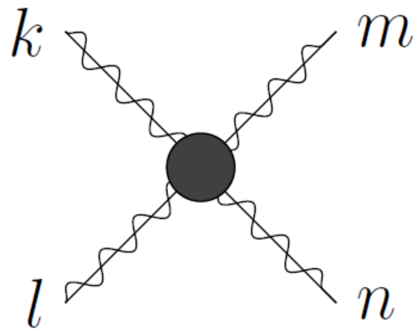


a_{lmn} $b_{l'm'n}$

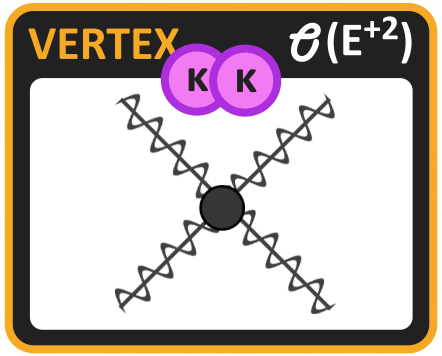


- Quartic Spin-2 Coupling -

h h h h



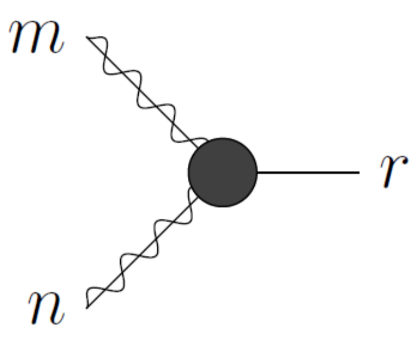
a_{klmn} $b_{k'l'mn}$



Relevant Interactions & Coupling Integrals

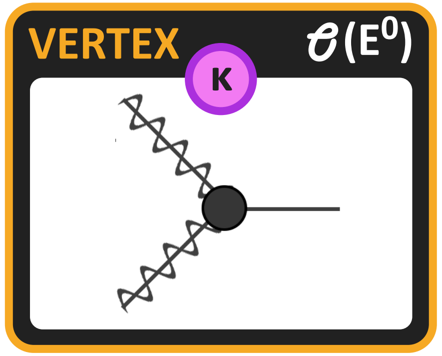
- Radion Coupling -

h h r



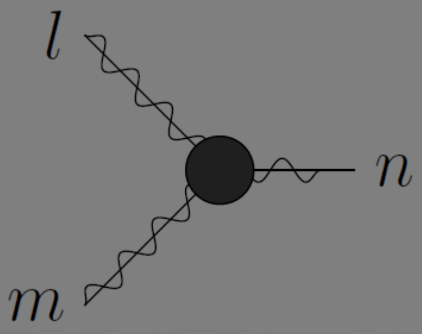
$b_{m'n'r}$

No A-Type

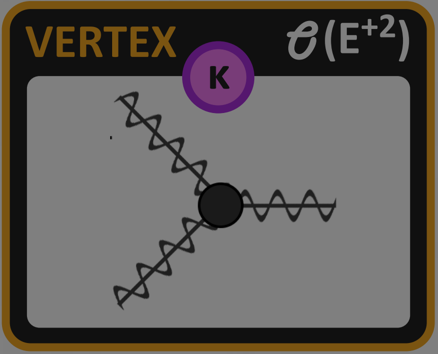


- Cubic Spin-2 Coupling -

h h h

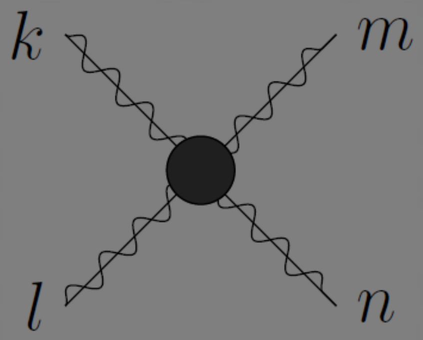


a_{lmn} $b_{l'm'n}$

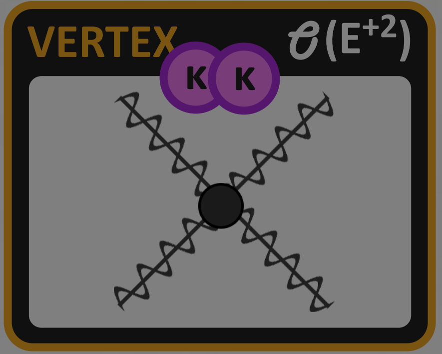


- Quartic Spin-2 Coupling -

h h h h

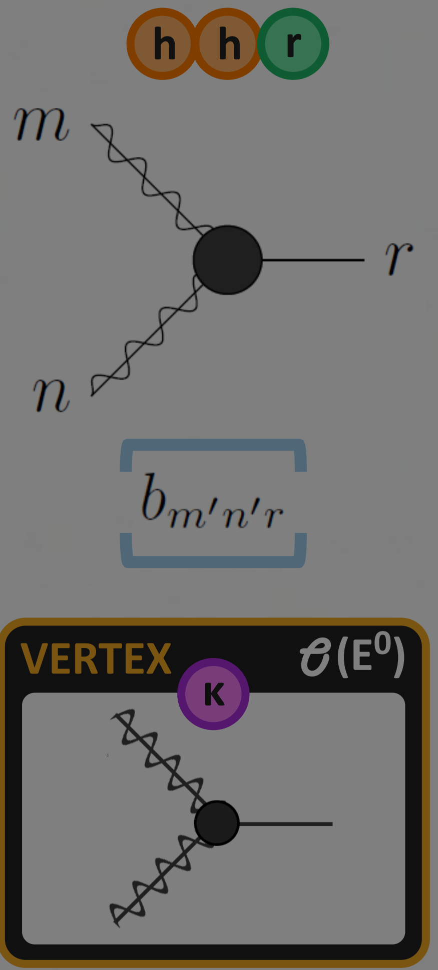


a_{klmn} $b_{k'l'mn}$

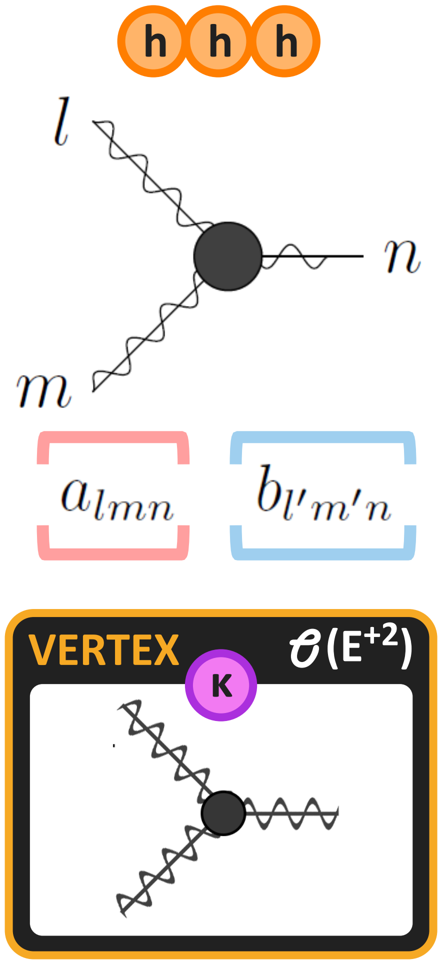


Relevant Interactions & Coupling Integrals

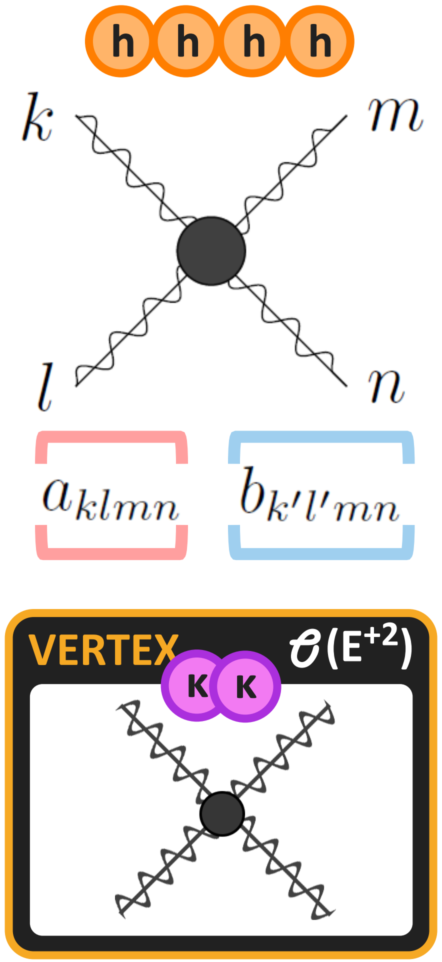
- Radion Coupling -



- Cubic Spin-2 Coupling -



- Quartic Spin-2 Coupling -



B-to-A Rules [PROVED]

$$\mu_n \equiv m_n r_c$$

[1910.06159]

- Elastic Case -

$$b_{n'n'j} = \frac{1}{2}\left[\mu_n^2 - \mu_j^2\right] a_{nnj}$$

$$b_{j'n'n} = \frac{1}{2}\mu_j^2 a_{nnj}$$

$$b_{n'n'nn} = \frac{1}{3}\mu_n^2 a_{nnnn}$$

B-to-A Rules [PROVED]

$$\mu_n \equiv m_n r_c$$

[1910.06159]

- Elastic Case -

$$b_{n'n'j} = \frac{1}{2}\left[\mu_n^2 - \mu_j^2\right] a_{nnj} \qquad b_{j'n'n} = \frac{1}{2}\mu_j^2 a_{nnj} \qquad b_{n'n'nn} = \frac{1}{3}\mu_n^2 a_{nnnn}$$

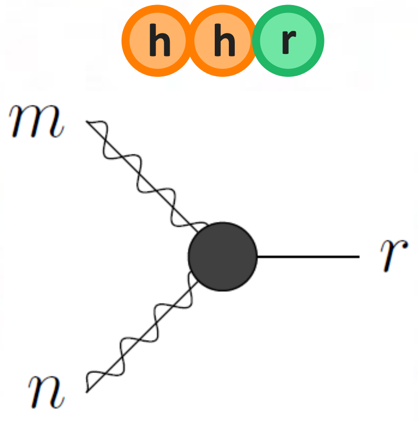
[Dissertation]

- Inelastic Generalization -

$$b_{l'm'n} = \frac{1}{2}\left[\mu_l^2 + \mu_m^2 - \mu_n^2\right] a_{lmn} \qquad b_{k'l'mn} = \frac{1}{6}\left[2\mu_k^2 + 2\mu_l^2 - \mu_m^2 - \mu_n^2\right] a_{klmn}$$

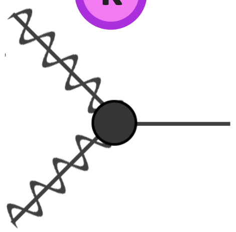
Relevant Interactions & Coupling Integrals

- Radion Coupling -

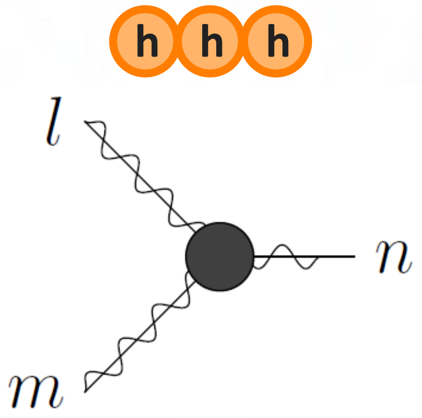


$b_{m'n'r}$

VERTEX K $\mathcal{O}(E^0)$

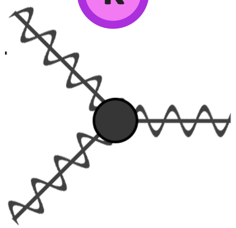


- Cubic Spin-2 Coupling -

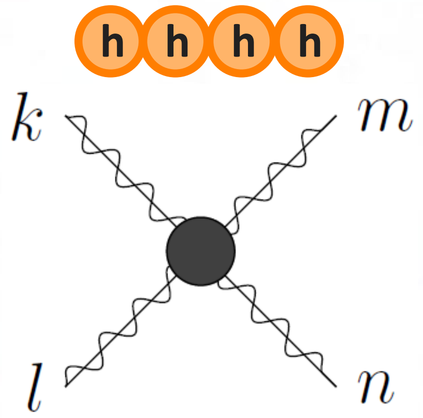


a_{lmn}

VERTEX K $\mathcal{O}(E^{+2})$

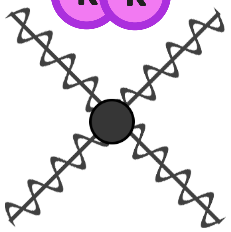


- Quartic Spin-2 Coupling -



a_{klmn}

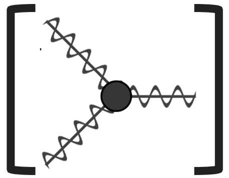
VERTEX K K $\mathcal{O}(E^{+2})$



Elastic Sum Rules [PROVED]

[1910.06159]
[Dissertation]

$$\mu_n \equiv m_n r_c$$



$$\Big[\ \cdot\ \Big]^2$$

$$\sum_j a_{jnn}^2 =$$

$$\sum_j \mu_j^2 a_{jnn}^2 =$$

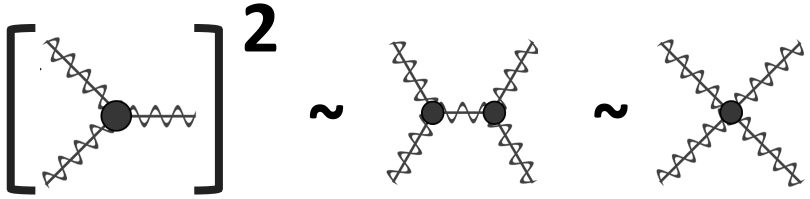
$$\sum_j \mu_j^4 a_{jnn}^2 =$$

$$\sum_j \mu_j^6 a_{jnn}^2 =$$

Elastic Sum Rules [PROVED]

[1910.06159]
[Dissertation]

$$\mu_n \equiv m_n r_c$$



$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

$$\sum_j \mu_j^4 a_{jnn}^2 = 4c_{nnnn} + \frac{4}{3}\mu_n^4 a_{nnnn}$$

$$\sum_j \mu_j^6 a_{jnn}^2 = 20\mu_n^2 c_{nnnn} + \frac{4}{3}\mu_n^6 a_{nnnn}$$

$$c_{nnnn} \equiv \frac{r_c^3}{\pi}\int_{-\pi r_c}^{+\pi r_c} dy \quad e^{-6k|y|}(\partial_y \psi_n)^4$$

Elastic Sum Rules [PROVED]

[1910.06159]
[Dissertation]

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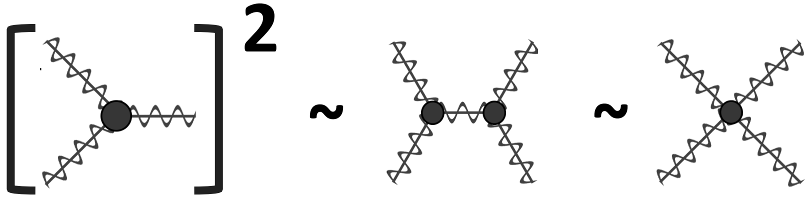

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$$\sum_j \left[\mu_j^2 - 5\mu_n^2\right]\mu_j^4 a_{jnn}^2 = -\frac{16}{3}\mu_n^6 a_{nnnn}$$

Inelastic Sum Rules [PROVED]

[Dissertation] $\mu_n \equiv m_n r_c$

$$\sum_j a_{jkl} a_{jmn} = a_{klmn}$$

$$\sum_j \mu_j^2 a_{jkl} a_{jmn} = \frac{1}{3}\vec{\mu}^{\,2} a_{klmn}$$

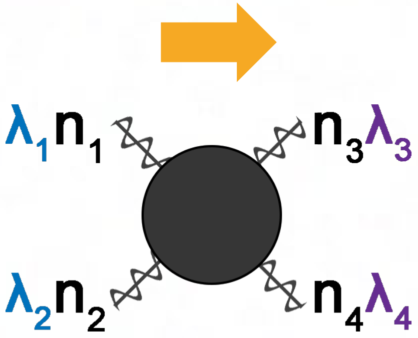
$$\sum_j \mu_j^4 a_{jkl} a_{jmn} = 4c_{klmn} + \left[\frac{1}{3}(\vec{\mu}^{\,2})^2 - (\mu_k^2 + \mu_l^2)(\mu_m^2 + \mu_n^2)\right] a_{klmn}$$

$$\sum_j \mu_j^6 a_{jkl} a_{jmn} = 5\vec{\mu}^{\,2} c_{klmn} - \frac{1}{9}\Big[6(\mu_k^4 + \mu_l^4)(\mu_m^2 + \mu_n^2) + 6(\mu_k^2 + \mu_l^2)(\mu_m^4 + \mu_n^4) - 4(\mu_k^2 + \mu_l^2)^3 - 4(\mu_m^2 + \mu_n^2)^3 + (\mu_k^6 + \mu_l^6 + \mu_m^6 + \mu_n^6)\Big] a_{klmn}$$

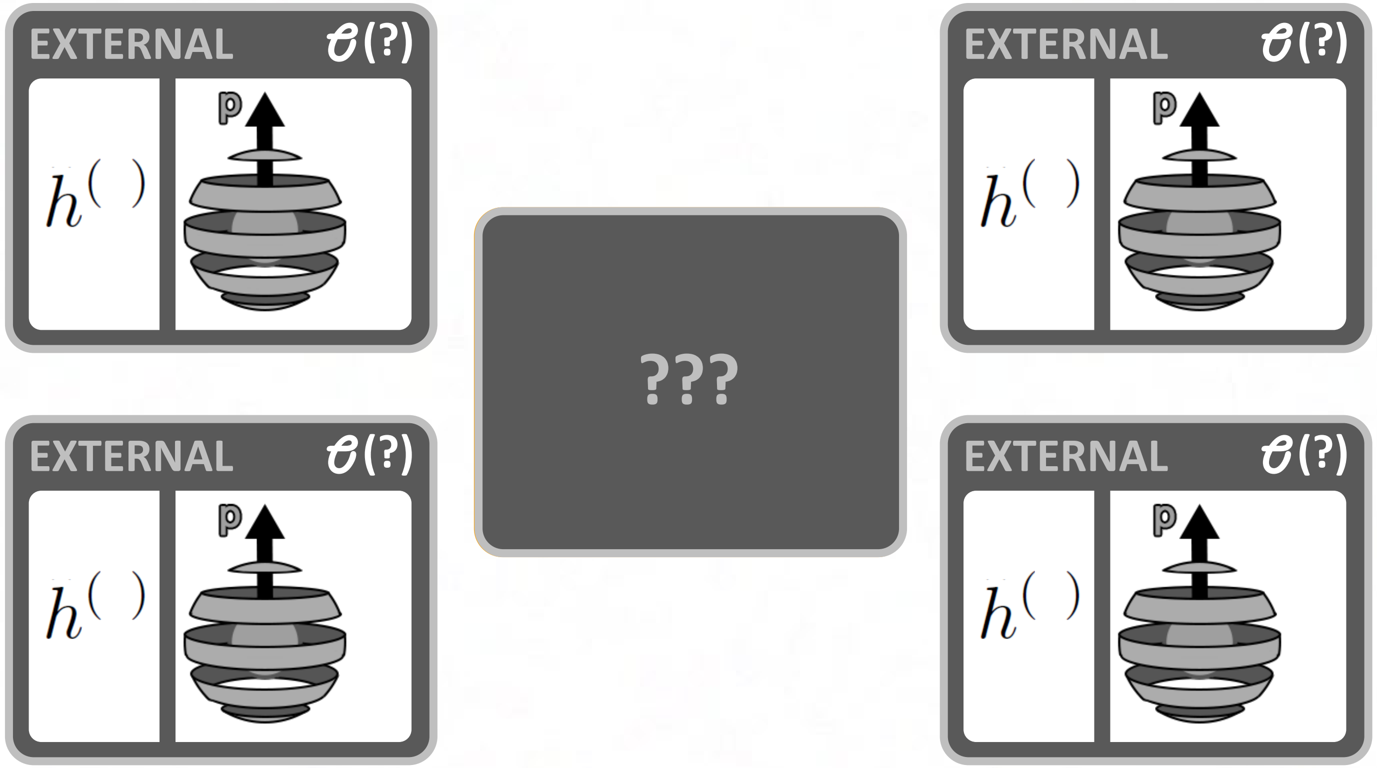
Matrix Elements

Matrix Element: General Considerations

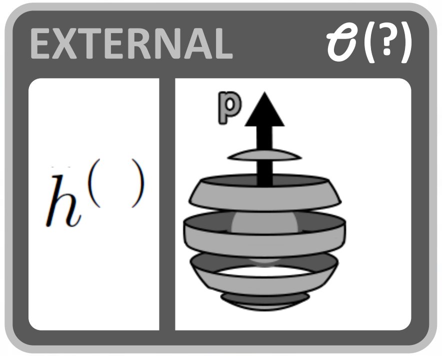
$\mathcal{M}_{\text{full}} =$



$=$



External States: Massive Spin-2 KK Modes



The ***n*th massive spin-2 KK mode** $\hat{h}^{(n)}_{\mu\nu}$ has **5 available helicities**

$$\lambda \in \{ -2 , -1 , 0 , +1 , +2 \}$$

External States: Massive Spin-2 KK Modes

The ***n*****th massive spin-2 KK mode** $\hat{h}^{(n)}_{\mu\nu}$ has **5 available helicities**

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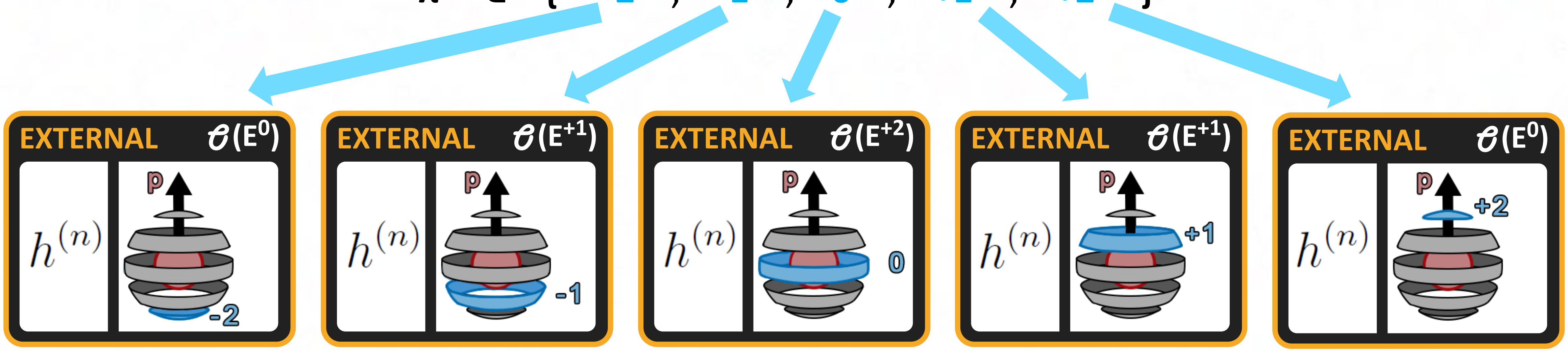
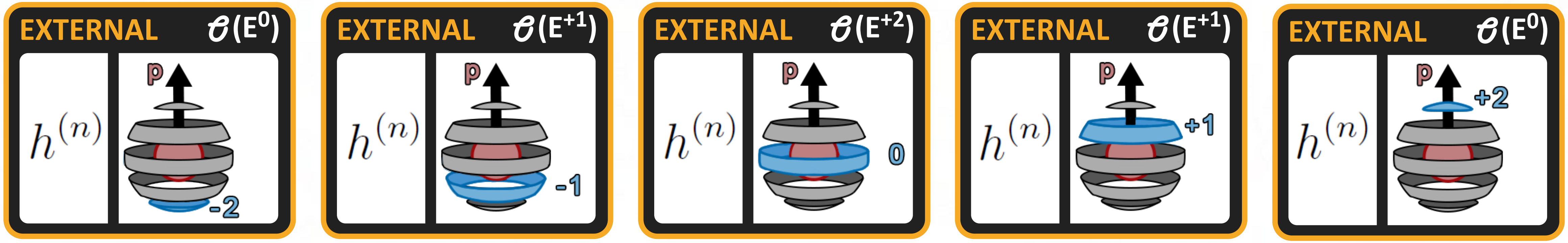


$\mathcal{O}(E^{0})$ $\mathcal{O}(E^{+1})$ $\mathcal{O}(E^{+2})$ $\mathcal{O}(E^{+1})$ $\mathcal{O}(E^{0})$

External States: Massive Spin-2 KK Modes

The ***n*th massive spin-2 KK mode** $\hat{h}^{(n)}_{\mu\nu}$ has 5 available helicities

$$\lambda \in \{ \; -2 \; , \; -1 \; , \; 0 \; , \; +1 \; , \; +2 \; \}$$

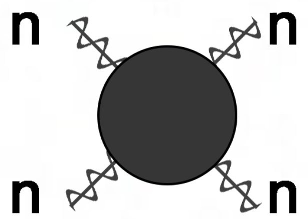


$\mathcal{O}(E^0)$ $\mathcal{O}(E^{+1})$ $\mathcal{O}(E^{+2})$ $\mathcal{O}(E^{+1})$ $\mathcal{O}(E^0)$

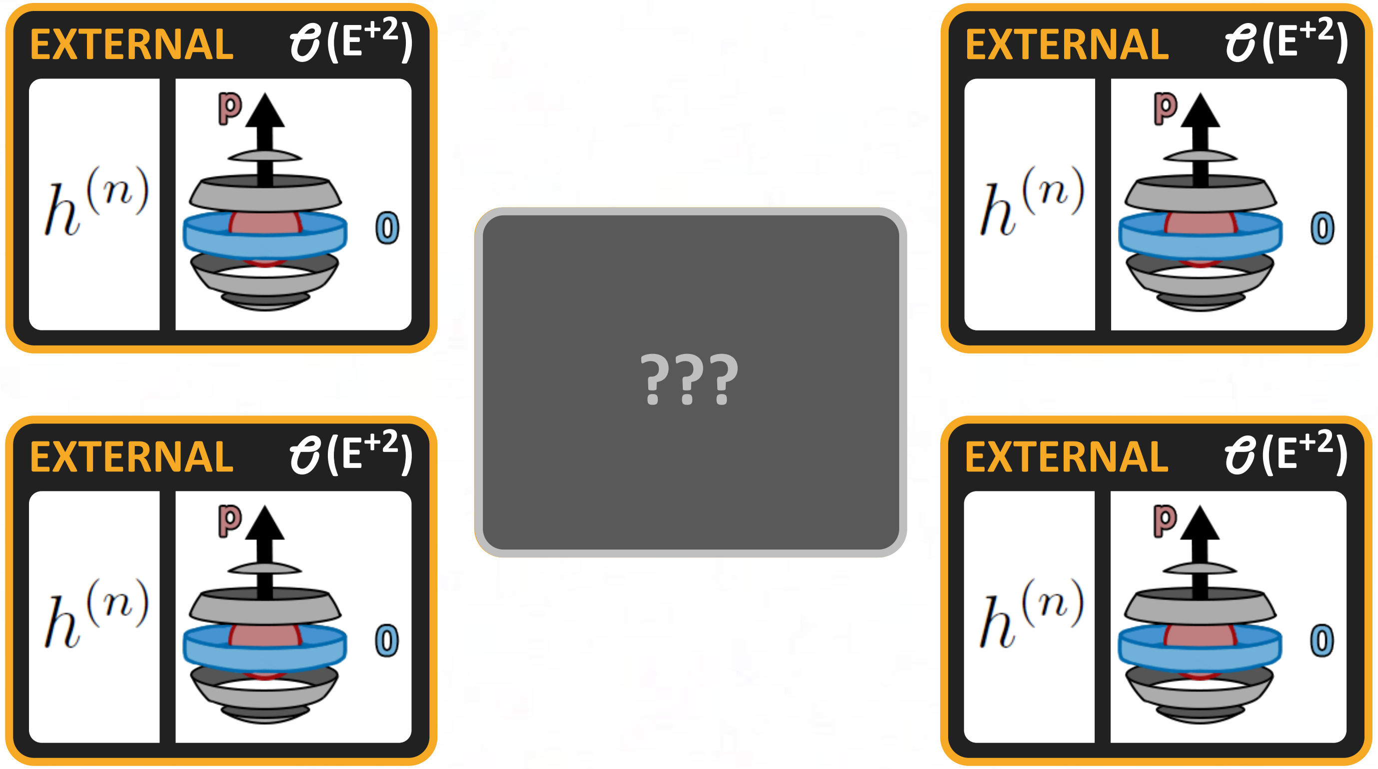
Fastest Growth in Energy
when **λ = 0**

Matrix Element: Elastic Helicity-Zero Process

$$\mathcal{M}_{\text{full}} = $$



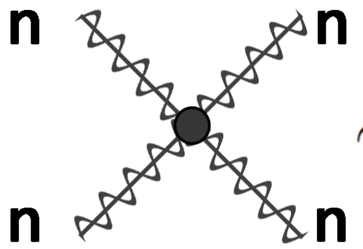
$$=$$



Contact: Elastic Helicity-Zero Process

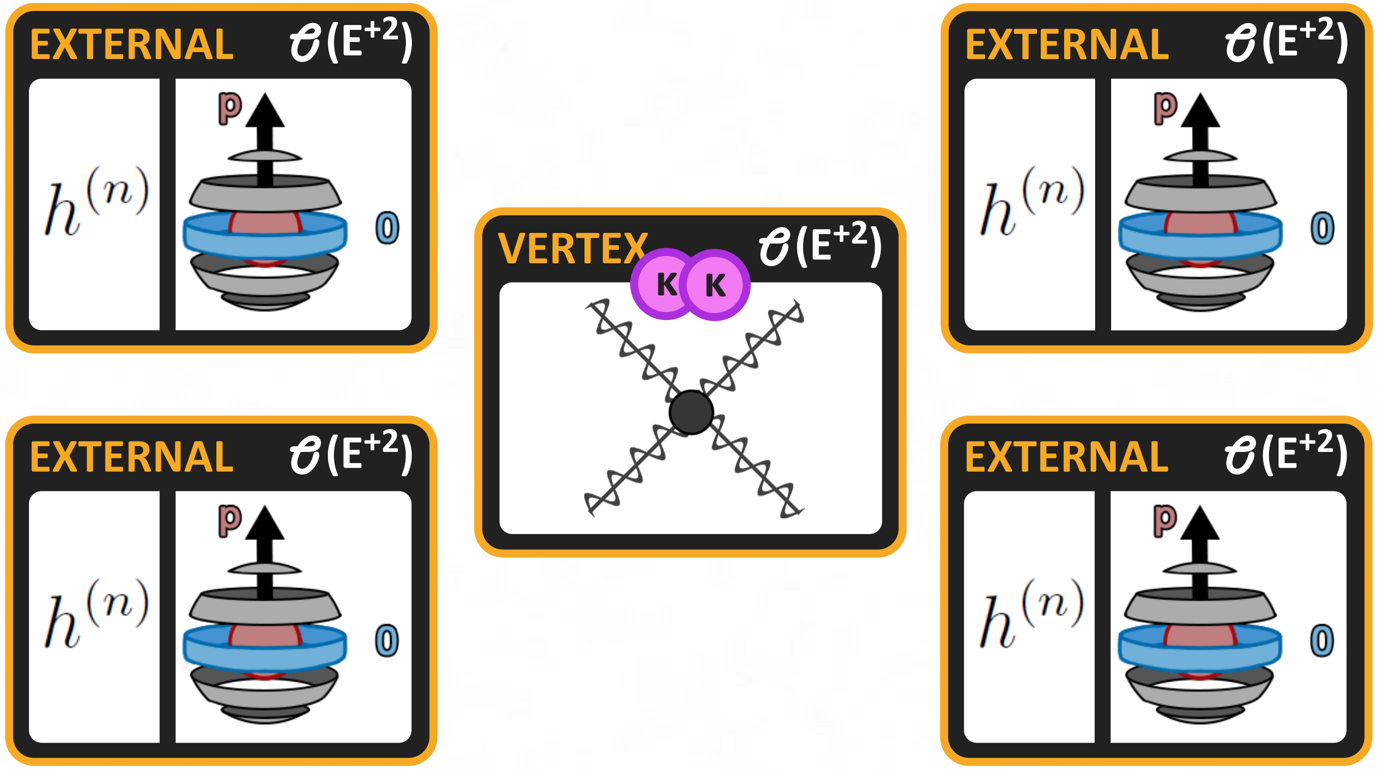
Contact Diagram

$$\mathcal{M}_c =$$

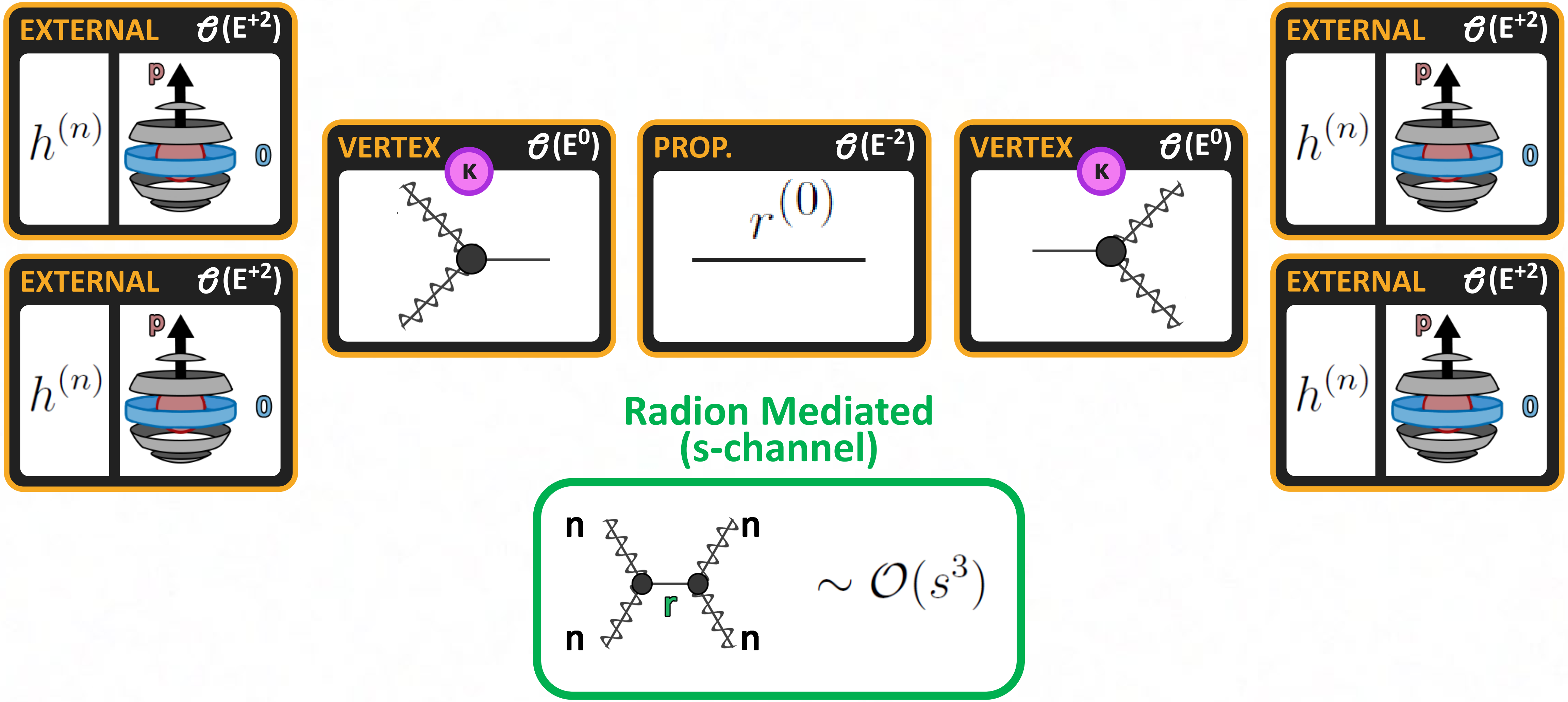


$$\sim \mathcal{O}(s^5)$$

[1906.11098] [2002.12458]

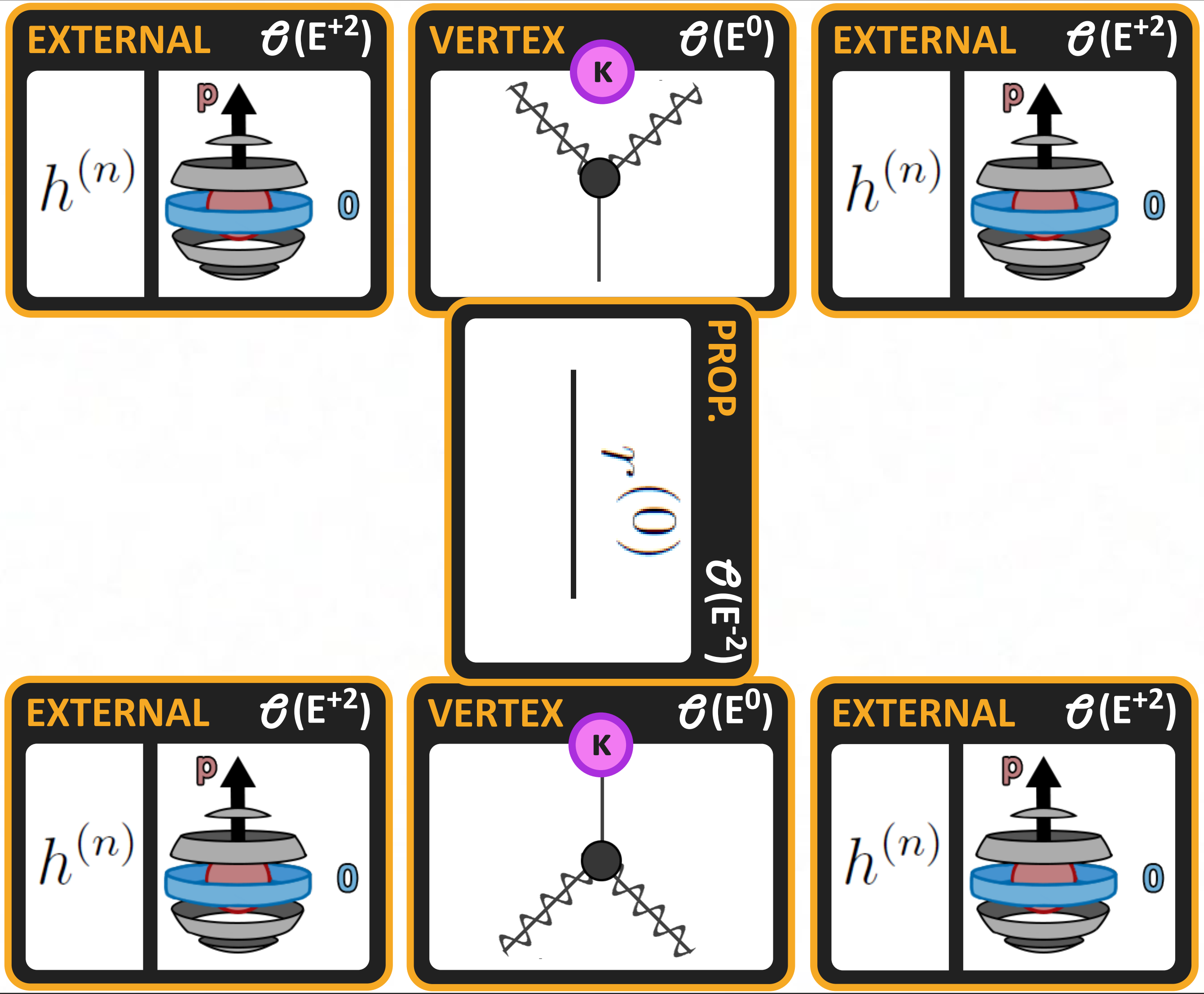


Radion Mediated: s - channel

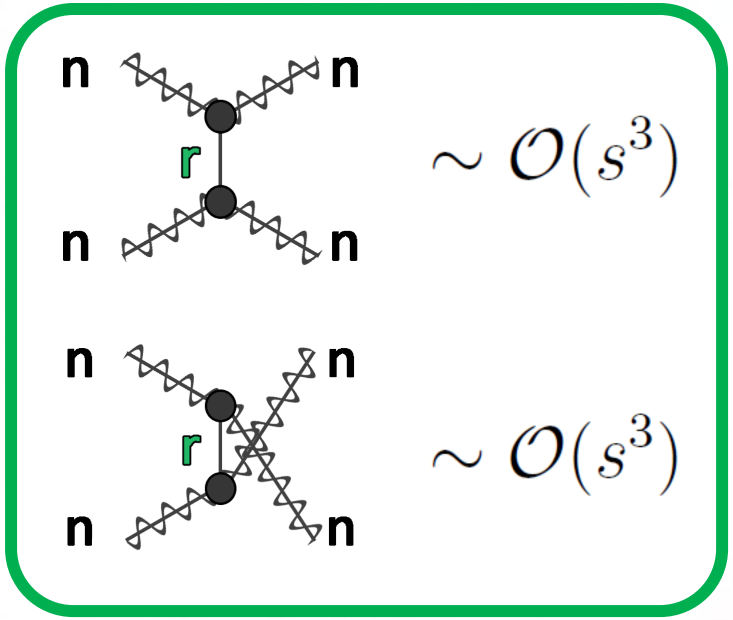


[1906.11098] [2002.12458]

Radion Mediated: t - and u - channel

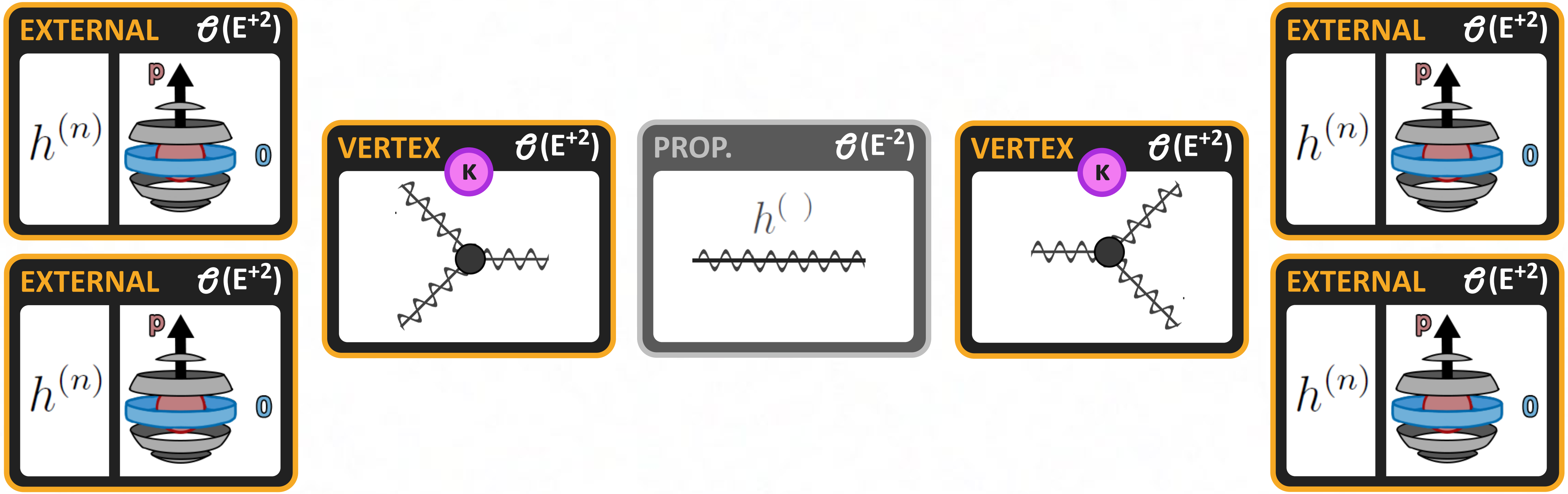


Radion Mediated (t- and u-channel)

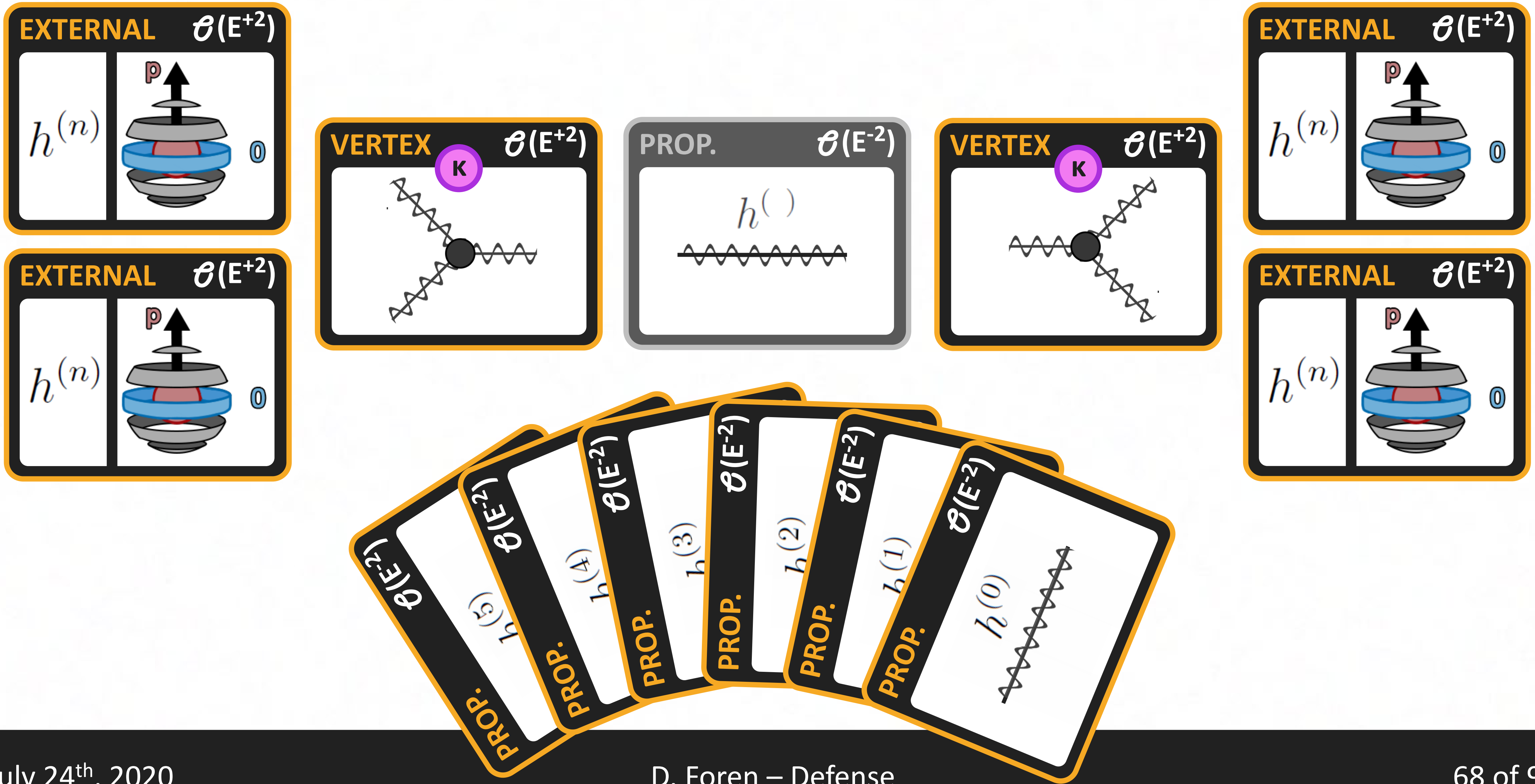


[1906.11098] [2002.12458]

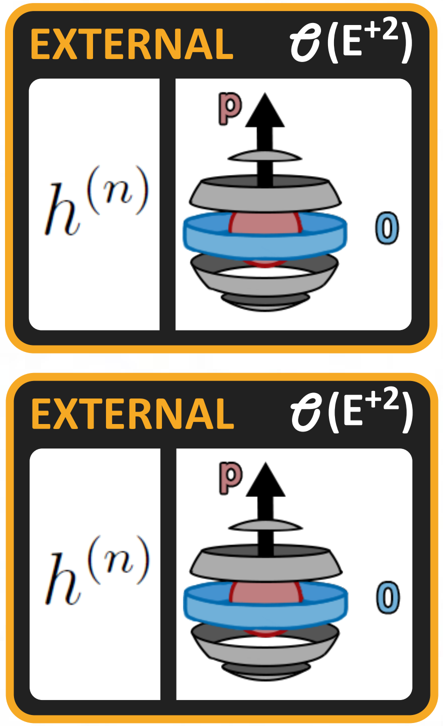
Spin-2 Mediated: s - channel

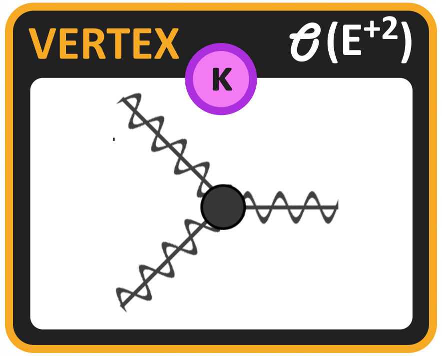


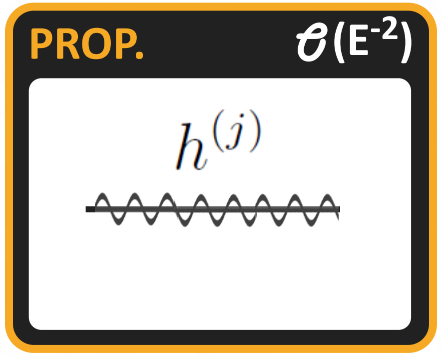
Spin-2 Mediated: s - channel

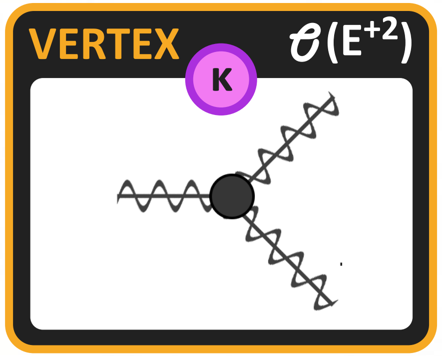


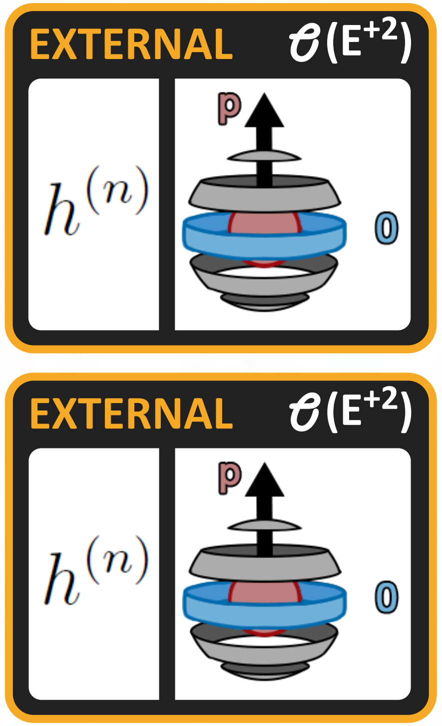
Spin-2 Mediated: s - channel



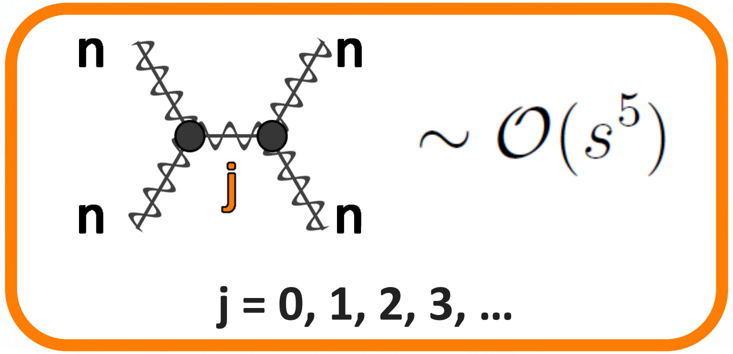








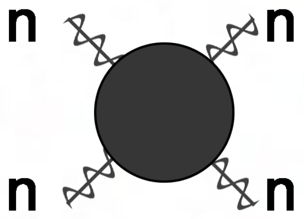
Spin-2 Mediated (s-channel)



[1906.11098] [2002.12458]

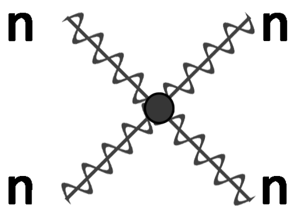
Matrix Element: Total Matrix Element

$$\mathcal{M}_{\text{full}} =$$

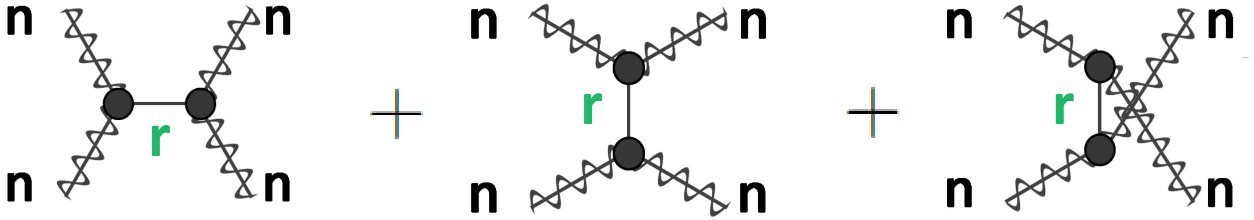


$$= \mathcal{M}_c + \mathcal{M}_r + \sum_{j=0}^{+\infty} \mathcal{M}_j$$

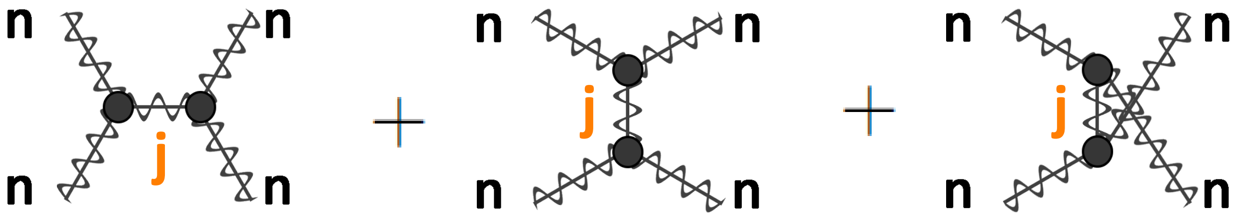
$$\mathcal{M}_c =$$



$$\mathcal{M}_r =$$

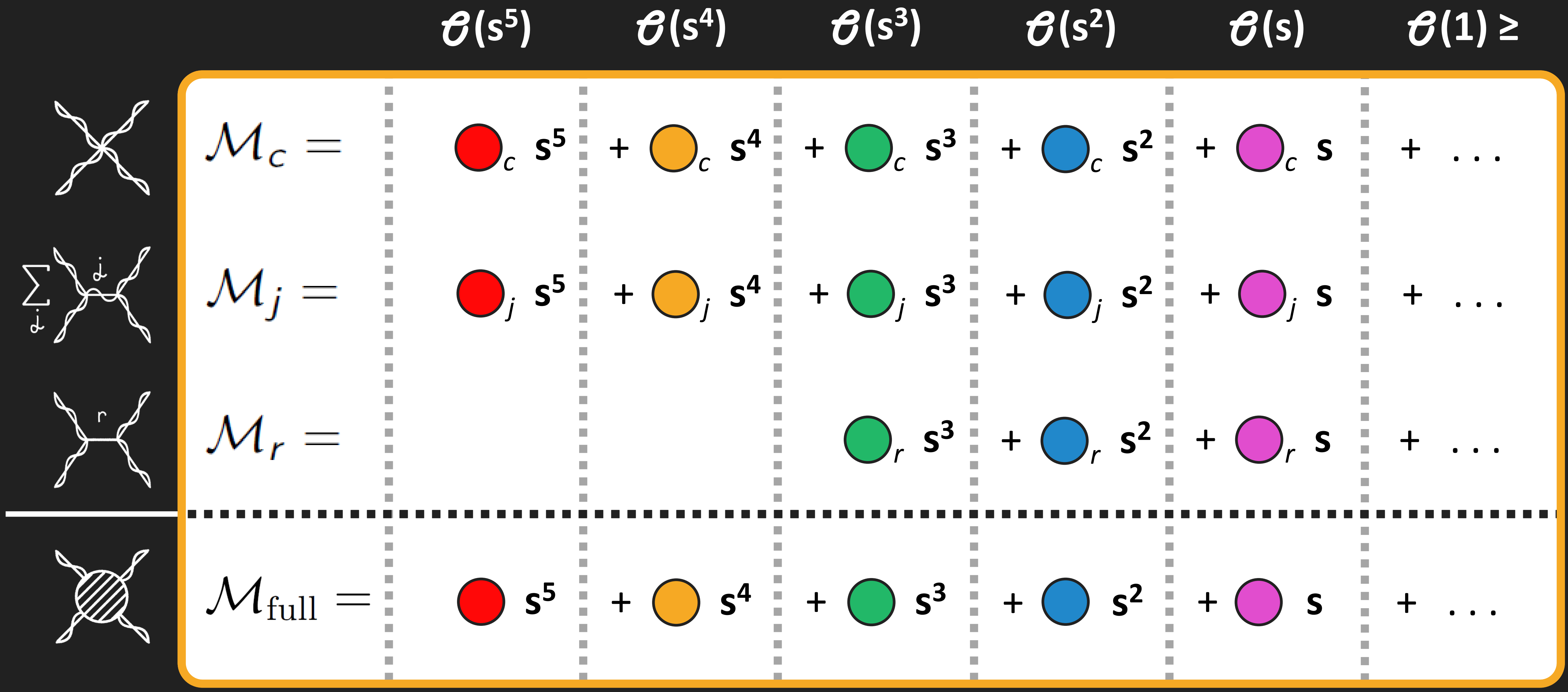


$$\mathcal{M}_j =$$



Helicity-Zero Elastic Process: Cancellations

[1906.11098]
[1910.06159]
[2002.12458]



Helicity-Zero Elastic Process: Cancellations

[1906.11098]
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[2002.12458]

Helicity-Zero Elastic Process: Cancellations

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Helicity-Zero Elastic Process: Cancellations

[1906.11098]
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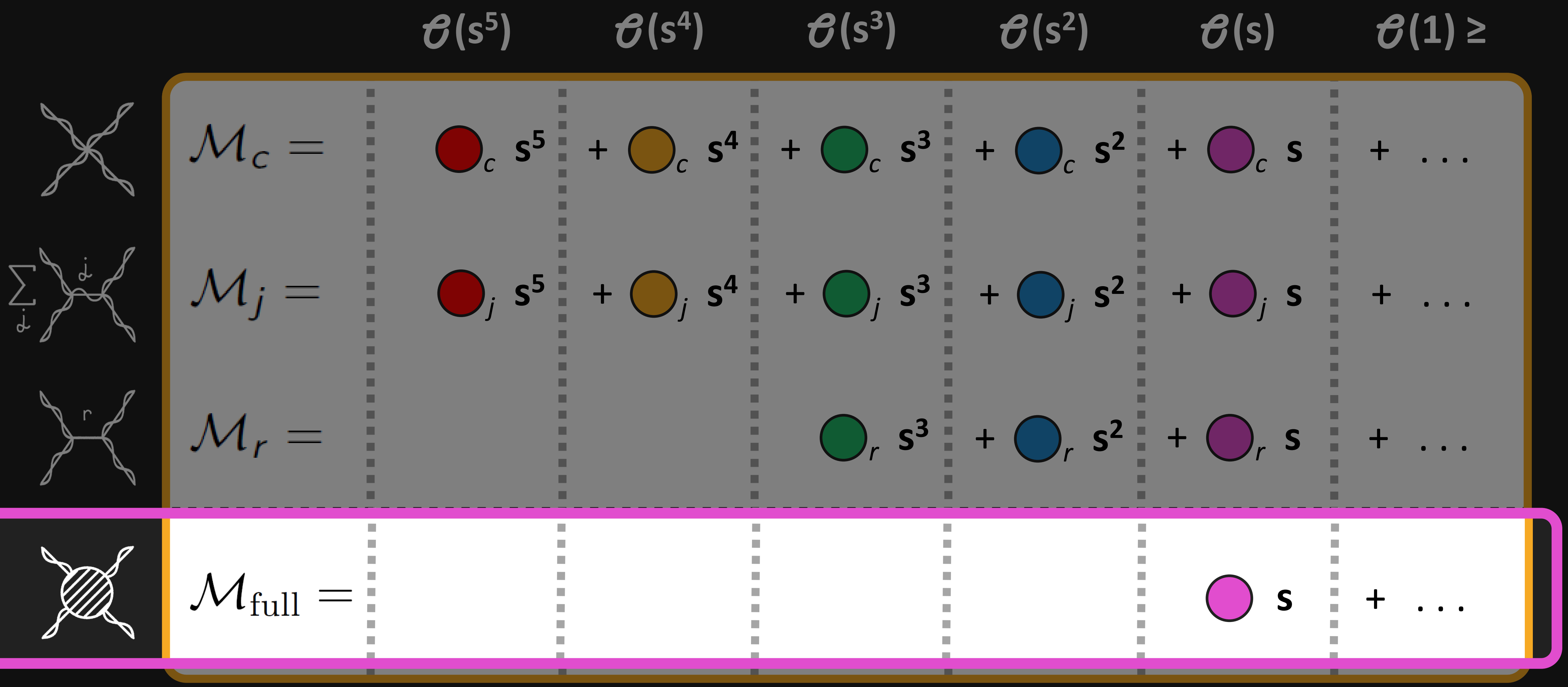
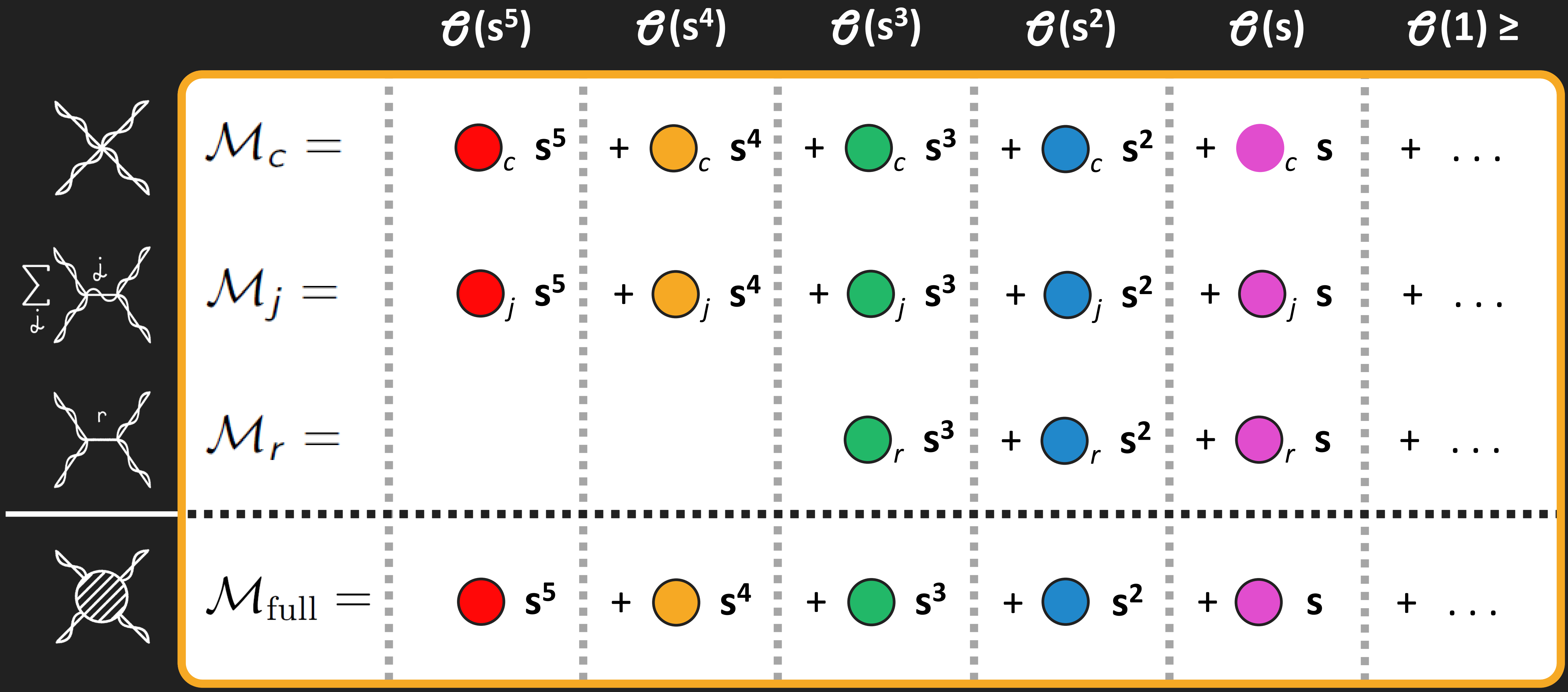


Helicity-Zero Elastic Process: Cancellations

[1906.11098]
[1910.06159]
[2002.12458]



Helicity-Zero Elastic Process: Cancellations

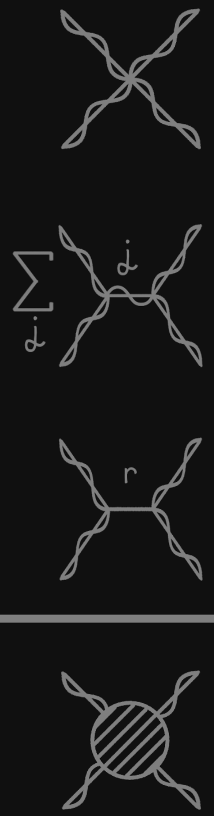
[1906.11098]
[1910.06159]
[2002.12458]

Helicity-Zero Elastic Process: Cancellations

[1906.11098]
[1910.06159]
[2002.12458]

Derivation of the $\mathcal{O}(s^5)$ Sum Rule

$\mathcal{O}(1) \geq$



$$\mathcal{M}_c = \bullet_c = -\frac{\kappa^2\, a_{nnnn}}{2304\,\pi r_c\, m_n^8}\,[7+\cos(2\theta)]\sin^2\theta + \ldots$$

$$\mathcal{M}_j = \bullet_j = \frac{\kappa^2\, a_{nnj}^2}{2304\,\pi r_c\, m_n^8}\,[7+\cos(2\theta)]\sin^2\theta + \ldots$$

$$\mathcal{M}_r = \qquad + \ldots$$

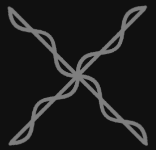
$$\mathcal{M}_{\text{full}} = \bullet = \frac{\kappa^2\,[7+\cos(2\theta)]\sin^2\theta}{2304\,\pi r_c\, m_n^8}\left\{\sum_{j=0}^{+\infty} a_{nnj}^2 - a_{nnnn}\right\} + \ldots$$

Helicity-Zero Elastic Process: Cancellations

[1906.11098] [1910.06159] [2002.12458]

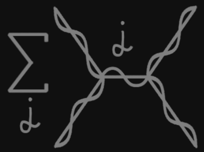
Derivation of the $\mathcal{O}(s^5)$ Sum Rule

$\mathcal{O}(1) \geq$



$\mathcal{M}_c =$

$$\bullet_c = -\frac{\kappa^2 a_{nnnn}}{2304\,\pi r_c m_n^8}\left[7+\cos(2\theta)\right]\sin^2\theta \quad + \;\ldots$$



$\mathcal{M}_j =$

$$\bullet_j = \frac{\kappa^2 a_{nnj}^2}{2304\,\pi r_c m_n^8}\left[7+\cos(2\theta)\right]\sin^2\theta \quad + \;\ldots$$

$\mathcal{M}_r = \quad + \;\ldots$

$\mathcal{M}_{\text{full}} =$

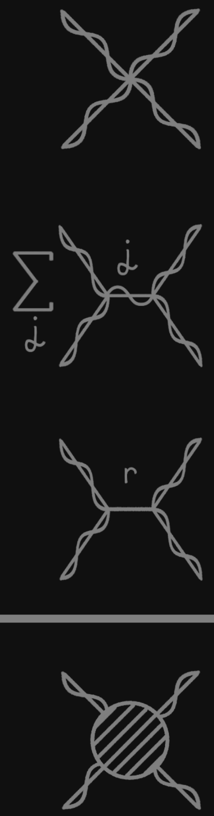
$$\bullet = \frac{\kappa^2\left[7+\cos(2\theta)\right]\sin^2\theta}{2304\,\pi r_c m_n^8}\left\{\sum_{j=0}^{+\infty} a_{nnj}^2 - a_{nnnn}\right\} \quad + \;\ldots$$

$$\Rightarrow \quad \sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}(s^5)$ Sum Rule

Helicity-Zero Elastic Process: Cancellations

[1906.11098]
[1910.06159]
[2002.12458]



	$\mathcal{O}(s^5)$	$\mathcal{O}(s^4)$	$\mathcal{O}(s^3)$	$\mathcal{O}(s^2)$	$\mathcal{O}(s)$	$\mathcal{O}(1) \geq$
$\mathcal{M}_c =$	$●_c\ s^5$	$●_c\ s^4$	$●_c\ s^3$	$●_c\ s^2$	$+\ ●_c\ s$	$+\ \ldots$
$\mathcal{M}_j =$	$●_j\ s^5$	$●_j\ s^4$	$●_j\ s^3$	$●_j\ s^2$	$+\ ●_j\ s$	$+\ \ldots$
$\mathcal{M}_r =$			$●_r\ s^3$	$●_r\ s^2$	$+\ ●_r\ s$	$+\ \ldots$
$\mathcal{M}_{\text{full}} =$	$●\ s^5$	$●\ s^4$	$●\ s^3$	$●\ s^2$	$+\ ●\ s$	$+\ \ldots$

Helicity-Zero Elastic Process: Sum Rules

[1910.06159]

$\mathcal{O}(s^5)$ Sum Rule

$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}(s^4)$ Sum Rule

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

$\mathcal{O}(s^3)$ Sum Rule

$$\sum_{j=0}^{+\infty} \mu_j^4 a_{nnj}^2 = \frac{16}{15}\mu_n^4 a_{nnnn} + \frac{4}{5}\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

$\mathcal{O}(s^2)$ Sum Rule

$$\sum_{j=0}^{+\infty} \left[\mu_j^2 - \frac{5}{2}\mu_n^2\right] \mu_j^4 a_{nnj}^2 = -\frac{8}{3}\mu_n^6 a_{nnnn} + 2\mu_n^2\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

[1910.06159]

Helicity-Zero Elastic Process: Sum Rules

$\mathcal{O}$ (s⁵) Sum Rule

$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}$ (s⁴) Sum Rule

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

$\mathcal{O}$ (s³) Sum Rule

$$\sum_{j=0}^{+\infty} \mu_j^4 a_{nnj}^2 = \frac{16}{15}\mu_n^4 a_{nnnn} + \ldots$$

$$\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

$\mathcal{O}$ (s²) Sum Rule

$$\sum_{j=0}^{+\infty} \left[\mu_j^2 - \frac{5}{2}\mu_n^2\right] \mu_j^4 a_{nnj}^2 = -\frac{8}{3}\mu_n^6 a_{nnnn} + 2\mu \ldots$$

$$\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

Helicity-Zero Elastic Process: Sum Rules

[1910.06159]

$\mathcal{O}(s^5)$ Sum Rule

$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}(s^4)$ Sum Rule

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

$\mathcal{O}(s^3)$ - $\mathcal{O}(s^2)$ Sum Rule

$$\sum_j \left[\mu_j^2 - 5\mu_n^2\right] \mu_j^4 a_{jnn}^2 = -\frac{16}{3}\mu_n^6 a_{nnnn}$$

$\mathcal{O}(s^2)$ Sum Rule

$$\sum_{j=0}^{+\infty} \left[\mu_j^2 - \frac{5}{2}\mu_n^2\right] \mu_j^4 a_{nnj}^2 = -\frac{8}{3}\mu_n^6 a_{nnnn} + 2\mu_n^2 \left[9 b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

Helicity-Zero Elastic Process: Sum Rules

[1910.06159]
[2002.12458]

$\mathcal{O}(s^5)$ Sum Rule

PROVED

$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}(s^4)$ Sum Rule

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

PROVED

$\mathcal{O}(s^3)$ - $\mathcal{O}(s^2)$ Sum Rule

$$\sum_j \left[\mu_j^2 - 5\mu_n^2\right]\mu_j^4 a_{jnn}^2 = -\frac{16}{3}\mu_n^6 a_{nnnn}$$

PROVED

$\mathcal{O}(s^2)$ Sum Rule

$$\sum_{j=0}^{+\infty}\left[\mu_j^2 - \frac{5}{2}\mu_n^2\right]\mu_j^4 a_{nnj}^2 = -\frac{8}{3}\mu_n^6 a_{nnnn} + 2\mu_n^2\left[9 b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right]$$

Helicity-Zero Elastic Process: Sum Rules

[1910.06159]
[2002.12458]
[Dissertation]

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PROVED

Simplified $\mathcal{O}(s^2)$ Sum Rule

$$3\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right] = 15c_{nnnn} + \mu_n^4 a_{nnnn}$$

Helicity-Zero Elastic Process: Sum Rules

[1910.06159]
[2002.12458]
[Dissertation]

$\mathcal{O}(s^5)$ Sum Rule

PROVED

$$\sum_j a_{jnn}^2 = a_{nnnn}$$

$\mathcal{O}(s^4)$ Sum Rule

$$\sum_j \mu_j^2 a_{jnn}^2 = \frac{4}{3}\mu_n^2 a_{nnnn}$$

PROVED

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$$\sum_j \left[\mu_j^2 - 5\mu_n^2\right] \mu_j^4 a_{jnn}^2 = -\frac{16}{3}\mu_n^6 a_{nnnn}$$

PROVED

Simplified $\mathcal{O}(s^2)$ Sum Rule

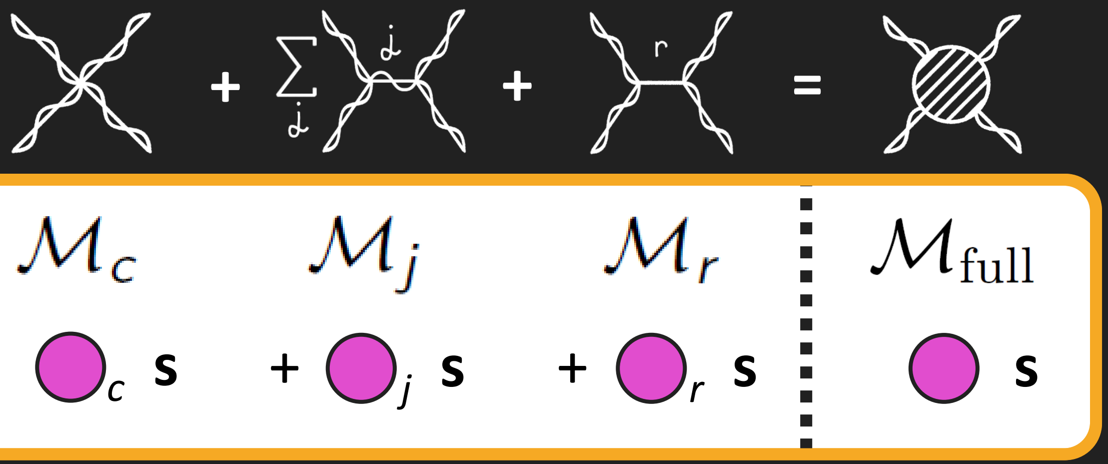
$$3\left[9b_{n'n'r}^2 - \mu_n^4 a_{nn0}^2\right] = 15c_{nnnn} + \mu_n^4 a_{nnnn}$$

IN PROGRESS

[1906.11098]
[1910.06159]
[2002.12458]

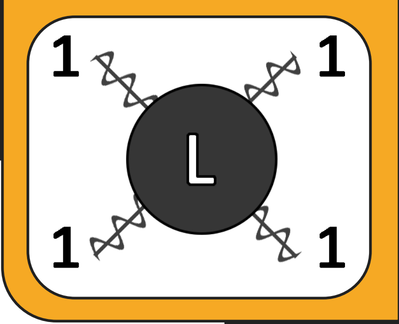
Helicity-Zero Elastic Process: O(s) Growth

$\mathcal{O}(s)$



$$\overline{\mathcal{M}}^{(1)} = \frac{\kappa^2\,[7+\cos(2\theta)]^2\csc^2\theta}{2304\,\pi r_c}\left\{\sum_j \frac{m_j^8}{m_n^8}a_{nnj}^2 + \frac{28}{15}a_{nnnn} - \frac{48}{5}\left[\frac{9\,b_{n'n'r}^2}{(m_n r_c)^4} - a_{nn0}^2\right]\right\}$$

Helicity-Zero (1,1) → (1,1) : Cancellations

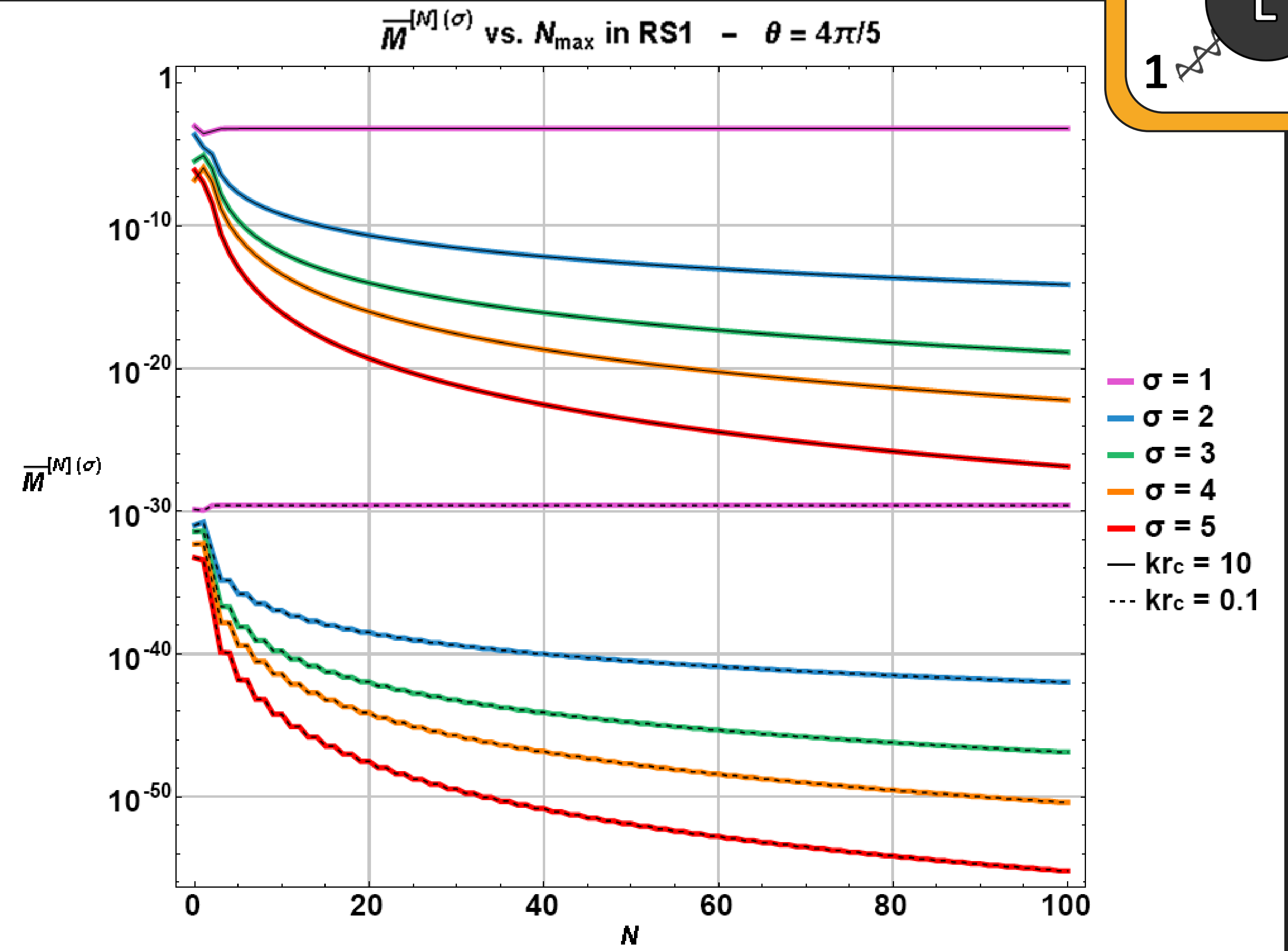


$$\mathcal{M}^{[N]} = \sum_{\sigma} \overline{\mathcal{M}}^{[N](\sigma)} \, (E r_c)^{2\sigma}$$

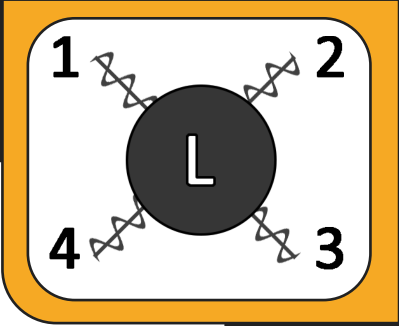
$$\mathcal{M}^{[N]} \equiv (\text{1st } N \text{ KK Modes})$$

Q: How does including more terms in the truncated sum affect leading energy growth?

A: By increasing N, any energy growth faster than O(E^2) increasingly cancels. O(E^2) remains.

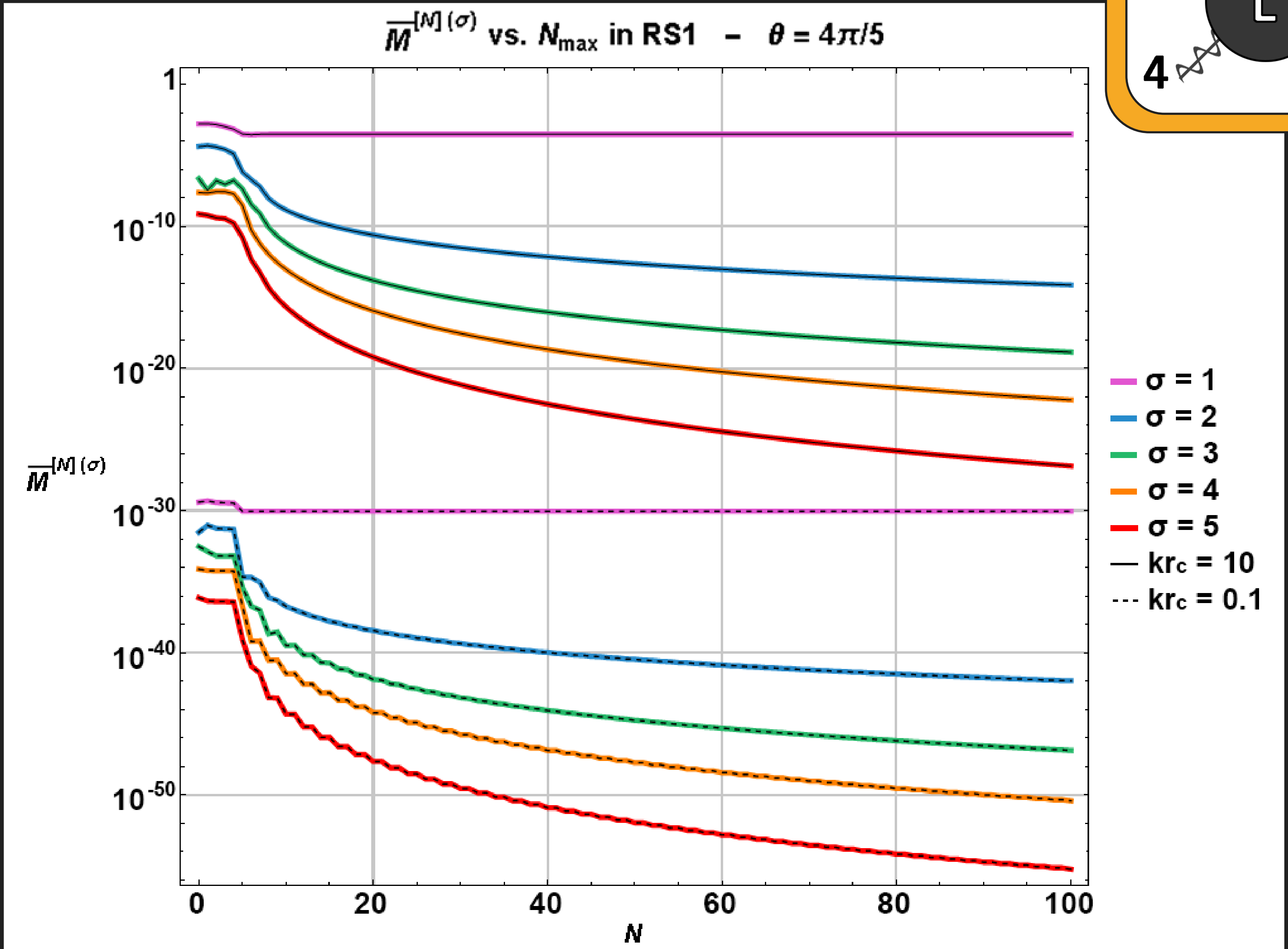


Helicity-Zero (1,4) → (2,3) : Cancellations



$$\mathcal{M}^{[N]} = \sum_{\sigma} \overline{\mathcal{M}}^{[N](\sigma)} \; (Er_c)^{2\sigma}$$

$$\mathcal{M}^{[N]} \equiv \; (\text{1st } N \text{ KK Modes})$$



Q: How does including more terms in the truncated sum affect leading energy growth?

A: By increasing N, any energy growth faster than O(E^2) increasingly cancels. O(E^2) remains.

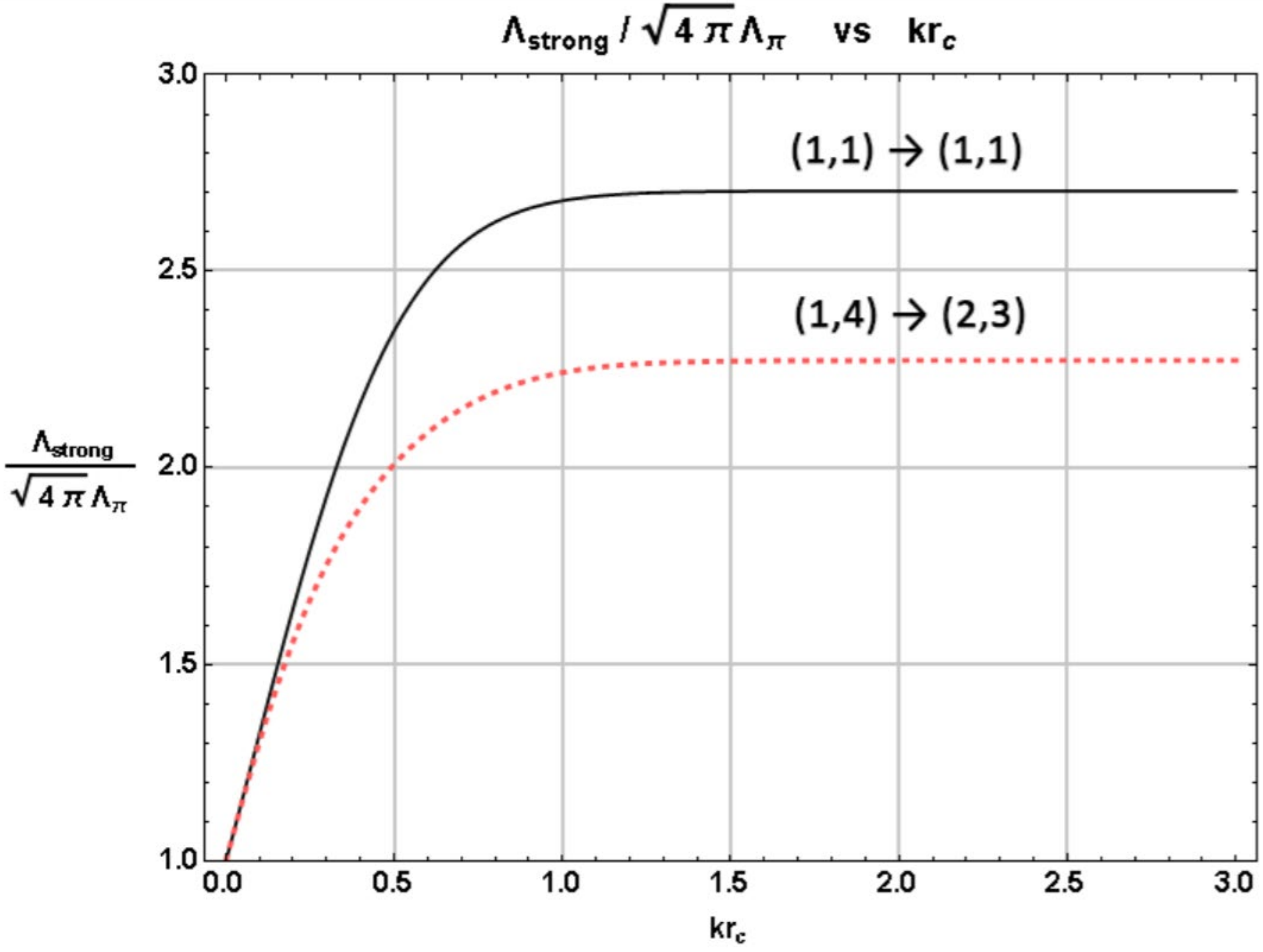
Strong Coupling Scale

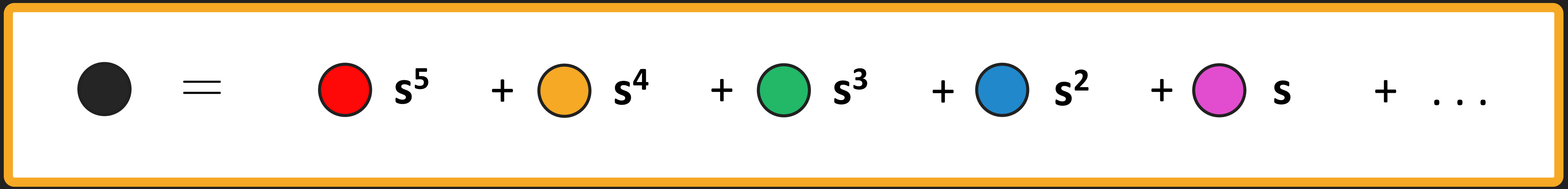
Is it consistent with expectations?

[2002.12458]

$$\Lambda_\pi \equiv M_{\rm Pl}\, e^{-\pi k r_c}$$

Strong Coupling Scale



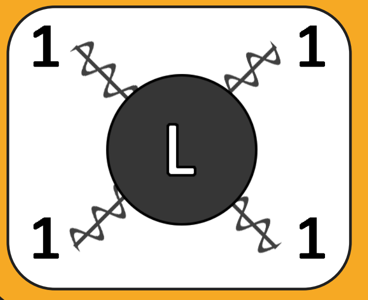


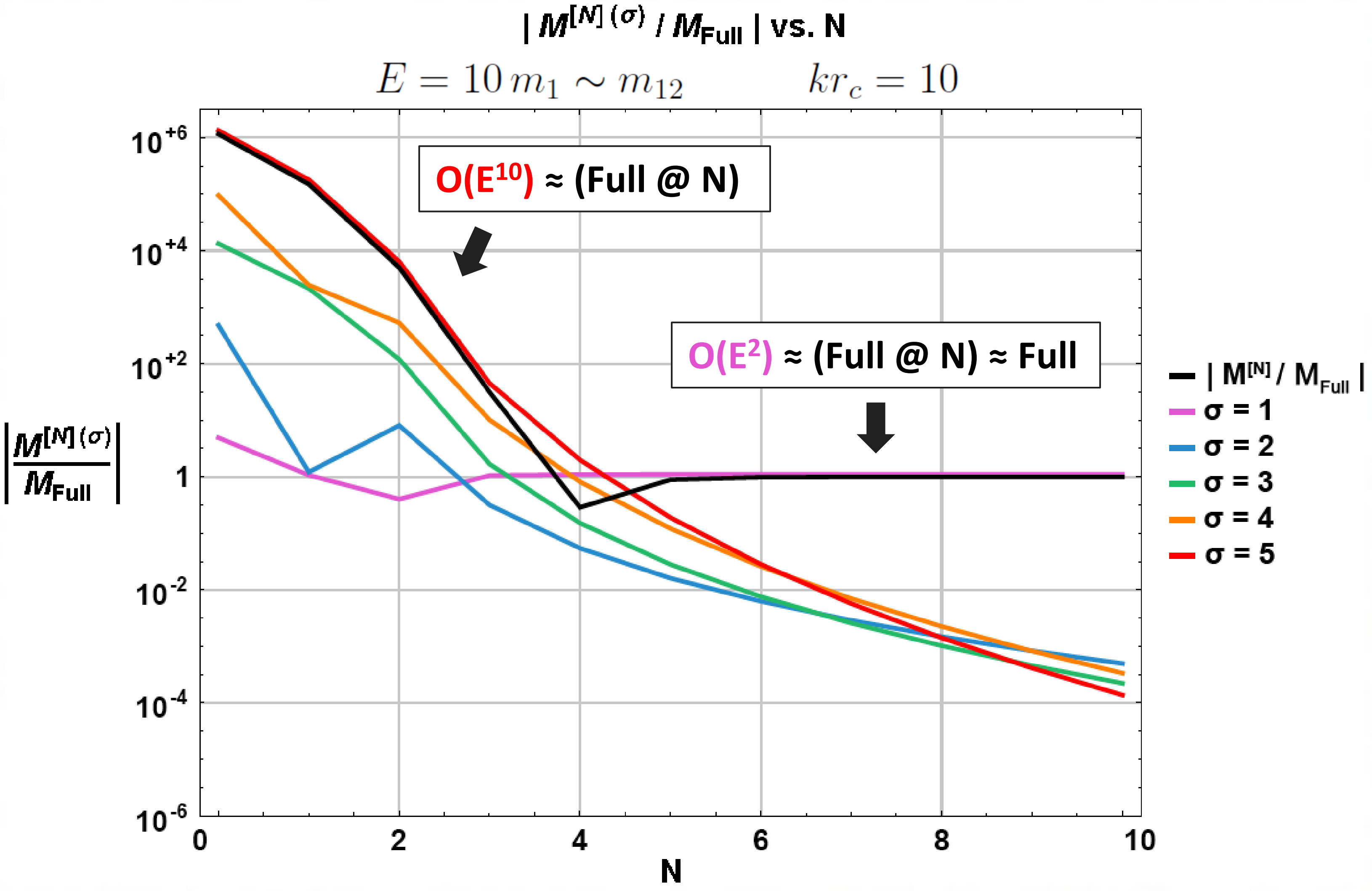
Finite Truncation at Low Energy

How many states should I include?

[2002.12458]

Truncation at Low Energies: $E = 10\ m_1$



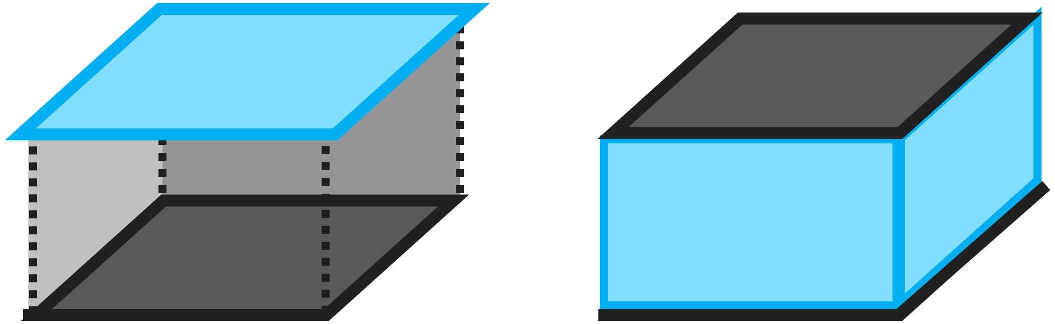


Conclusion

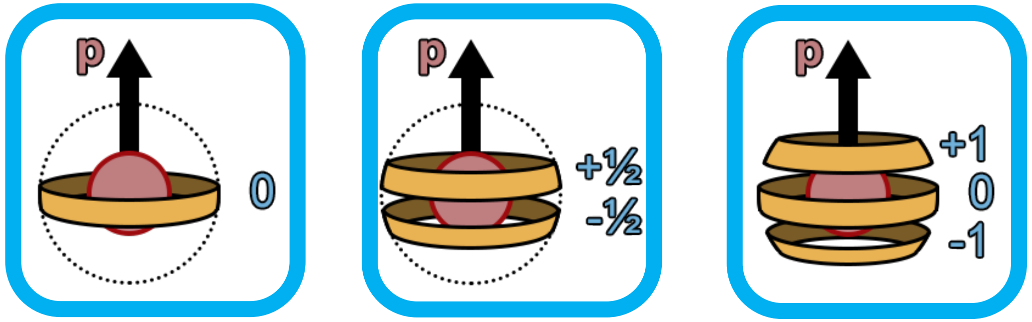
Future Work

- Bulk/Brane Matter -

add matter to...

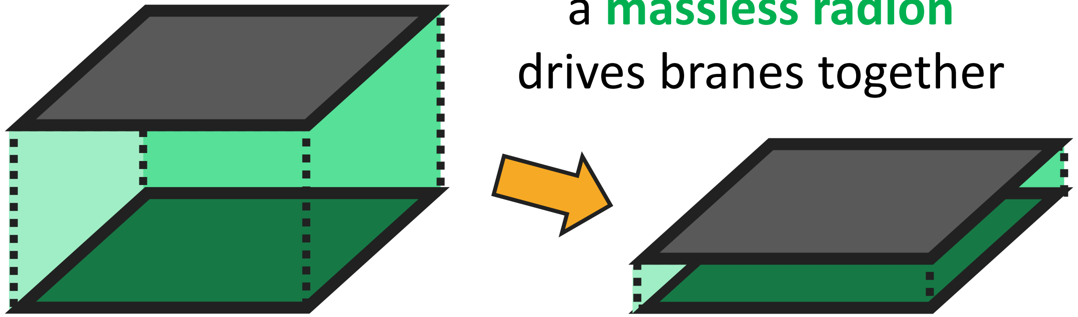


a brane or **the bulk**



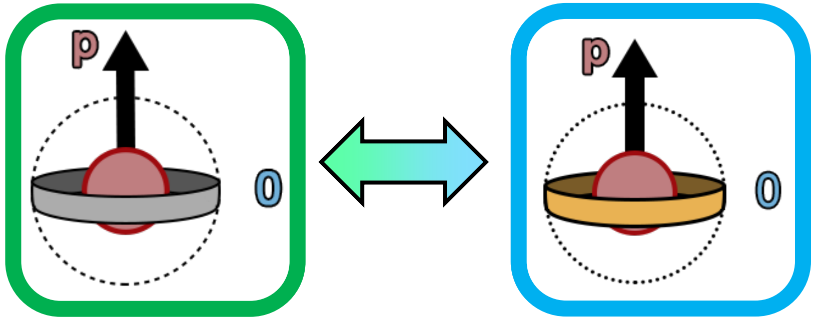
scalar **fermion** **vector**

- Radion Stabilization -



a **massless radion** drives branes together

Radion can be **stabilized** through mixing:



massless radion **massive bulk scalar**

Conclusion

- Further Reading -

"Scattering Amplitudes of Massive Spin-2 Kaluza-Klein States Grow Only as $\mathcal{O}(s)$"

[Phys. Rev. D 101, 055013] [arXiv:1906.11098]

"Sum Rules for Massive Spin-2 Kaluza-Klein Elastic Scattering Amplitudes"

[Phys. Rev. D 100, 115033] [arXiv:1910.06159]

"Massive Spin-2 Scattering Amplitudes in Extra-Dimensional Theories"

[Phys. Rev. D 101, 075013] [arXiv: 2002.12458]

"Scattering Amplitudes in Theories of Compactified Gravity"

[Dissertation]

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Thank you for your time and attention!